

Shadow lower-bound term structure models: Applications to Singapore

Leo Krippner
Sim Kee Boon Institute of
Singapore Management University
University of Waikato (New Zealand)

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Outline of presentation

Singapore context

- exchange rate is key
- why do interest rates matter?

Term structure models

- overview of standard model
- allowing for interest rate lower bound

Application: shadow rates

- why, what, and how for most central banks
- first pass results and context for Singapore

Application: yield decompositions

- why, what, and how for most central banks
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1. Singapore context

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Monetary policy in Singapore

How does MAS achieve price stability?

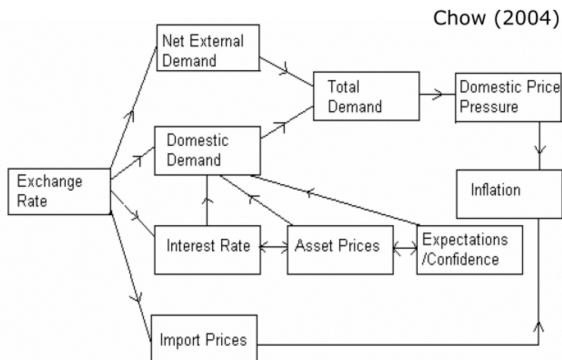
"In a small and open economy such as Singapore, where gross exports and imports of goods and services are more than 300 percent of GDP and domestic expenditure has a high import content, the exchange rate has a much stronger influence on inflation than the interest rate. Accordingly, MAS' monetary policy framework is centred on managing the Singapore dollar against a trade-weighted basket of currencies."

<https://www.mas.gov.sg/monetary-policy/singapores-monetary-policy-framework>

So why are we here to talk about interest rates and the yield curve?

Interest rates are part of monetary transmission

Figure 2. Monetary Transmission Mechanism

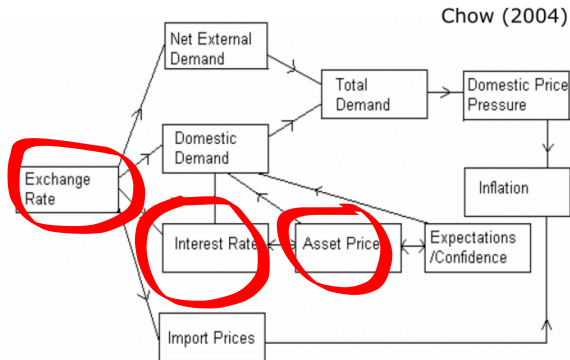


Note: This diagram is modified from MAS (1999) and for simplicity, does not show all interactions between variables.

**Although exchange rate \implies interest rates
(elsewhere interest rate \implies exchange rate)**

Interest rates & yield curve part of monetary conditions

Figure 2. Monetary Transmission Mechanism



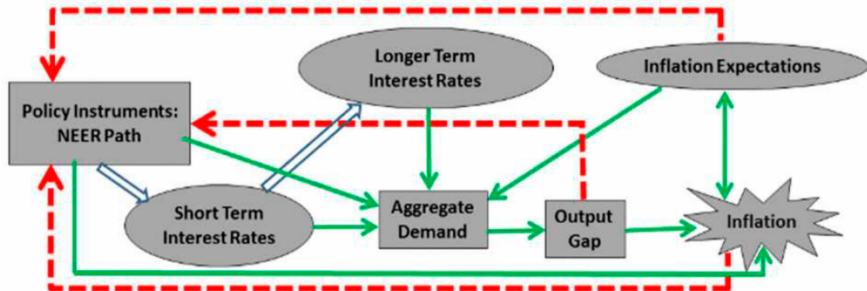
Note: This diagram is modified from MAS (1999) and for simplicity, does not show all interactions between variables.

But contribution to monetary conditions smaller for Singapore than for other economies

A figure explicitly including the yield curve

Figure 2. NEER-Based Model for an IT Regime

IMF (2016)



(asset prices didn't make the cut in this IMF figure)

Uncovered Interest Parity and exchange rate expectations

UIP (Uncovered Interest Parity) condition:

- $[3\text{-m IR }] + \mathbb{E}[3\text{-m } \Delta\text{NEER }] = [\text{foreign 3-m IR }]$
- **UIP actually holds for Singapore !!**
 - e.g. MAS (2000), MAS & Mihov (2013)
 - not the case for most (all?) other economies
- **likely reflects operation of monetary policy via exchange rate, since 1981, Mihov et. al (2017)**

UIP more generally, with re-arrangement:

- $\mathbb{E}[\text{path } \Delta\text{NEER }] = \mathbb{E}[\text{foreign IR path }] - \mathbb{E}[\text{IR path }]$

So interest rate paths imply an NEER path

Uncovered Interest Parity and exchange rate expectations

IN PRINCIPLE could monitor implicit $\mathbb{E}[\text{path } \Delta\text{NEER}]$ using market-quoted interest rates:

- data available at high frequency & real time
- complement to lower-frequency survey-based expectations

BUT subject to open questions and caveats:

- should IR paths be forward rates from yield curve?
 - mechanical calculation, or from model
 - but either includes risk/term premiums
- or should risk/term premiums be removed from IR path?
 - needs to be estimated from a model
- how to handle unconventional/lower-bound periods?

Will any such results be sensible and useful?

2. Term structure models

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Gaussian affine term structure models: easy as 1, 2, 3, 4 ...

1. Specify & solve Gaussian diffusion for “short rate”:



2. Specify risk adjustment:

3. Solve for interest rates by maturity:



4. Estimate GATSM parameters and variables:



Gaussian affine term structure models: ... but some math

1. Specify & solve Gaussian diffusion for “short rate”:

- **state variables under physical measure \mathbb{P} :**

$$dx(t) = \kappa [x(t) - \theta]dt + \sigma dW(t)$$

- **short rate:** $r(t) = a_0 + b'_0 x(t)$

2. Specify risk adjustment: $\Pi(t) = \sigma^{-1} [\gamma + \Gamma x(t)]$

3. Solve for interest rates by maturity:

- **state variables under risk-adjusted measure \mathbb{Q} :**

$$dx(t) = \tilde{\kappa} [x(t) - \tilde{\theta}]dt + \sigma d\tilde{W}(t)$$

- **forward rates:** $f(t, \tau) = b'_0 \exp(-\tilde{\kappa}\tau) x(t) + \text{VE}(\tau)$

- $\text{VE}(\tau) = \frac{1}{2} [\int_0^\tau b'_0 \exp(-\tilde{\kappa}u) du] \sigma \sigma' [\int_0^\tau \exp(-\tilde{\kappa}'u) b_0 du]$

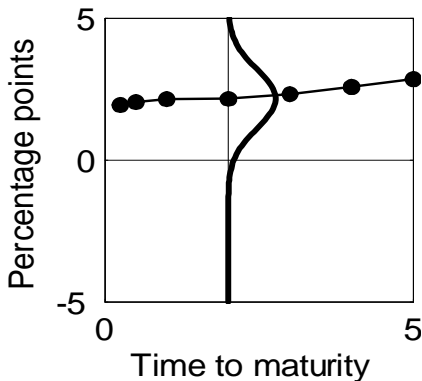
- **interest rates:** $R(t, \tau) = \frac{1}{\tau} \int_0^\tau f(t, u) du$ (closed form)

4. Estimate GATSM parameters and variables:

- standard Kalman filter to best fit yield curve data
- state equation $x(t)$, measurement equation $R(t, \tau)$

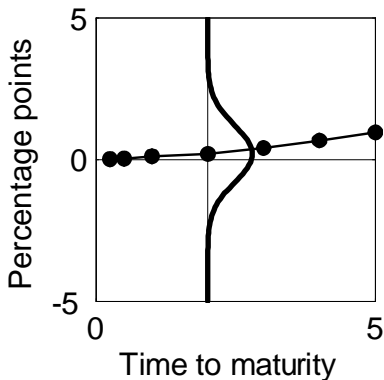
But Gaussian models have a (no floor) flaw

- interest rates can evolve to negative values
- physical currency offers an “interest rate” of zero
- Great arbitrage !! – But not “real world” consistent
- Small enough to ignore? **Maybe, in normal times ...**



... but not near interest rate lower bound !!

- **Gaussian models very mis-specified** relative to “real world” where interest rates are LB-constrained



My solution: use shadow term structure model to price bonds plus bond options to represent “option of physical currency”

Krippner (2011-15) shadow-LB framework

Steps 1-3: Use GATSM for shadow term structure, and then:

LB forward rates are (closed form):

$$\underline{f}(t, \tau) = f(t, \tau) \cdot \Phi \left[\frac{f(t, \tau)}{\omega(\tau)} \right] + \omega(\tau) \cdot \frac{1}{\sqrt{2\pi}} \exp \left(-\frac{1}{2} \left[\frac{f(t, \tau)}{\omega(\tau)} \right]^2 \right)$$

$f(t, \tau)$ is now shadow forward rate, $\Phi[\cdot]$ is cumulative normal function, and $[\omega(\tau)]^2 = \int_0^\tau b_0' \exp(-\tilde{\kappa}u) \sigma \sigma' \exp(-\tilde{\kappa}'u) b_0 du$

LB interest rates by univariate numerical integration:

$$\underline{R}(t, \tau) = \frac{1}{\tau} \int_0^\tau \underline{f}(t, u) du$$

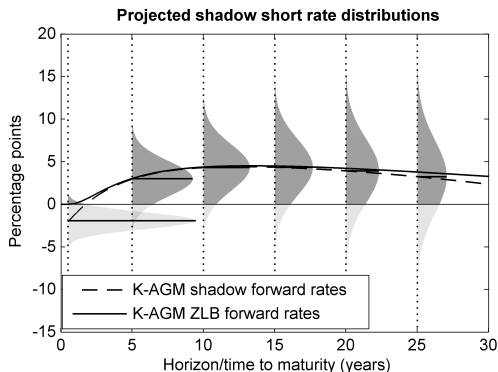
4. Estimate model parameters and variables

- non-linear Kalman filter with measurement equation $\underline{R}(t, \tau)$

Key figure for shadow-LB framework

$$\text{SSR distribution: } r(t + \tau | x(t)) \sim N\left\{f(t, \tau), [\omega(\tau)]^2\right\}$$

$$\text{LB forward rate: } \underline{f}(t, \tau) = f(t, \tau) + \tilde{\mathbb{E}}_{t+\tau}[\max\{r_L - r(t + \tau), 0\}]$$



Is shadow/LB framework still relevant?

Not now, but needed for consistent history (and future?)

Final math: state variables are Nelson-Siegel coefficients

SSR models use two-factor **Arbitrage-Free Nelson-Siegel**

- $x(t) = [L(t), S(t)]'$, Level and Slope
- $f_{LS}(t) = L(t) + S(t) \cdot \exp(-\phi\tau) + \text{VE}_{LS}(\tau)$
- this choice produces most robust and sensible SSRs
- using three factors, e.g. Wu and Xia (2015), is too flexible. Produces sensitive and non-sensible SSRs
- details in Krippner (2015, 2020), Krippner et al. (2018)

Yield decomposition models use standard three-factor **AFNS**

- $x(t) = [L(t), S(t), B(t)]'$, adds Bow/Curvature
- $f_{LSB}(t) = f_{LS}(t) + B(t) \cdot \phi\tau \exp(-\phi\tau) + \text{VE}_{LSB}(\tau)$

AFNS is a popular model within GATSM class

3. Application: shadow rates

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Conventional monetary policy (CMP) with interest rates

In CMP times, central bank **Policy Interest Rate (PIR)** & closely related short-maturity rates played many roles, e.g:

- 1 primary instrument of MP (what central banks control)
- 2 indicator of “the stance of MP”
(broadly, MP stimulus to the real economy and inflation)
- 3 MP variable used in macroeconomic forecasting and analysis, as part of CB seeking its targets

Conventional monetary policy



Policy Interest Rate (PIR) was the **instrument** and **metric** for Monetary Policy

Unconventional Monetary Policy (UMP)

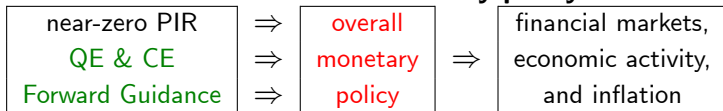
In addition to the near-zero PIR, central bank also uses a mix of other policy instruments, e.g.:

- **Quantitative Easing (QE) & Credit Easing (CE)**: involving asset purchase, liquidity, and bank lending programmes
- **Forward Guidance**: statements of PIR intentions with stronger hints of unconditionality (as compared to typical CMP policy rate indications, like PIR tracks conditional on economic forecasts)
- **Negative PIRs**: arguably CMP extension, but often included because historically unprecedented

PIR alone will no longer indicate the stance of MP

So what is the stance of UMP?

Unconventional monetary policy



One approach: **quantify UMP with yield curve data**

- yield curve: interest rates for different times to maturity
- principle is yield curve embeds current PIR & UMP settings, and guidance/expectations of their evolution

Shadow Short Rates (SSRs) estimated from yield curve data are one example

Leo's 15 minutes of fame

- Speech by St. Louis Fed President, November 2012:

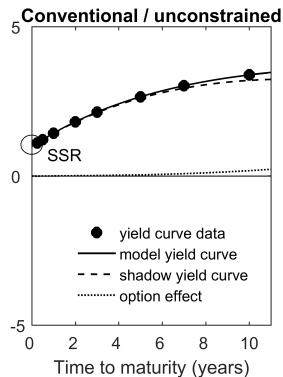
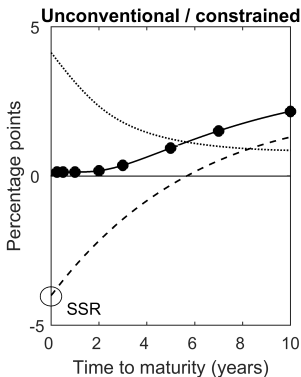
Wall Street Journal: *Relying on the work of Leo Krippner of the central bank of New Zealand, Mr. Bullard said this new thinking suggests the policies now being pursued by the Fed leave the funds rate as equivalent to a negative-5% rate.*

Reuters: *Bullard highlighted research by Leo Krippner, an economist at the Reserve Bank of New Zealand, who studied the option value of cash when interest rates are zero, to illustrate just how accommodative U.S. policy may be at the moment.*

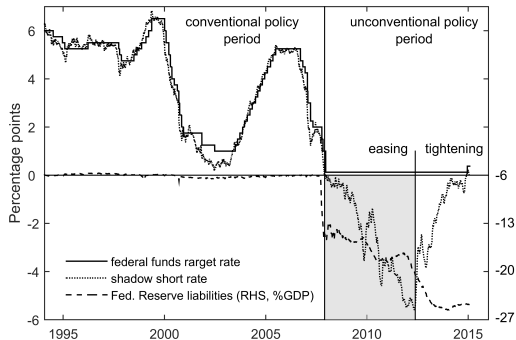
- 2017 Economics in Central Banking Award
- Footnote in ECB President Draghi speech, June 2019

SSR estimate is an output from shadow/LB model

- **LB short rate = shadow short rate + currency option**
 - $\underline{r}(t) = r(t) + \max[-r(t), r_L]$, (re-arranged from Black 1995)
- \Rightarrow **LB yields = shadow yields + option effect**



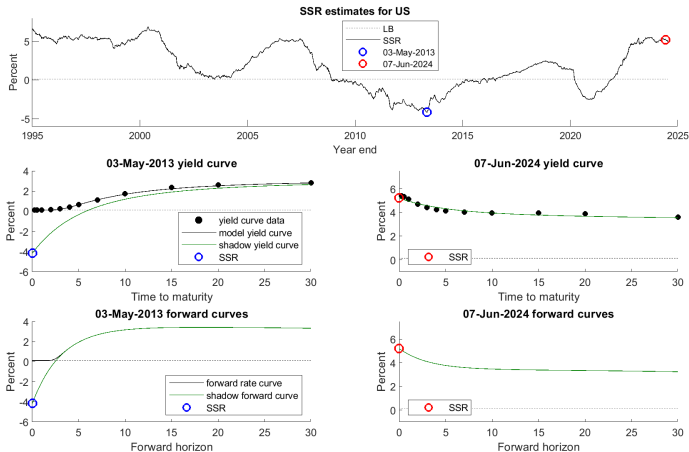
Example of SSR estimates for the US



Negative SSR indicates stimulus beyond near-zero PIR:

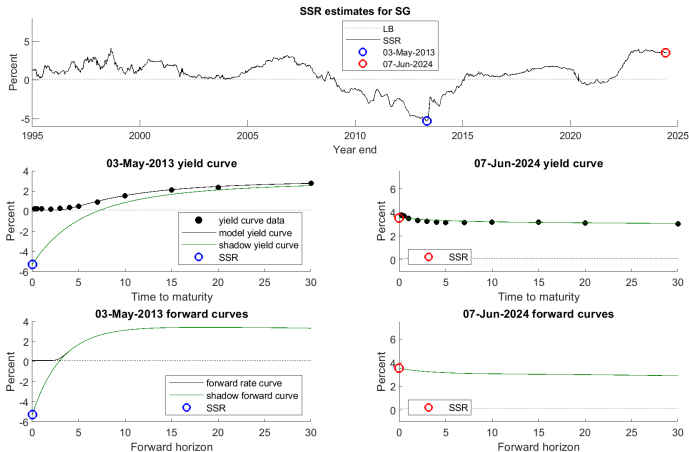
- note: -ve & **falling** SSR during balance sheet expansion
- note: -ve & **rising** SSR as balance sheet expansion ends
- Forward Guidance also an influence, e.g. “taper tantrum”

Updated SSR estimates for United States



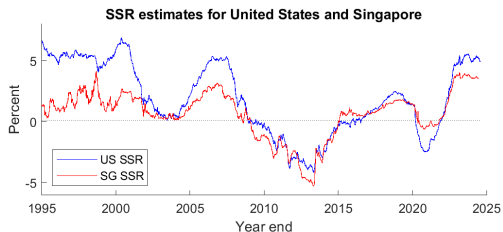
Now includes COVID-era UMP

Preliminary SSR estimates for Singapore



Uses Singapore yield curve data with US parameters

Context for negative SSRs in Singapore



- MAS didn't implement UMP measures (I think?)
- but Singapore "imports" interest rates from NEER basket countries, less allowance for $\mathbb{E}[\text{path } \Delta\text{NEER}]$, via UIP
- USD arguably has high weight (direct & indirect trade links)
- so US yield curve highly influences Singapore yield curve
- and Singapore SSR closely correlates with the US

Interest rate channel for Singapore provided more stimulus during GFC & COVID than near-zero market rates

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Decomposition of a bond yield

Bond yield for a given maturity contains two components:

- **EPIR**: Expected Policy Interest Rate component
- **BYRP**: Bond Yield Risk Premium component

EPIR reflects rolling investment in PIR path to bond maturity

BYRP reflects “**everything but EPIR**”:

- fundamental risk/return characteristics of bonds
 - how bonds hedge consumption path under uncertainty
 - tangible proxy is diversification benefits from bonds within a portfolio, e.g. correlation of bond/equity returns
- non-fundamental but important non-EPIR influences, e.g.:
 - potential loss/recovery on default
 - liquidity/safety benefits during financial market stress
 - effects of QE/QT and other interventions

Basis for the decomposition? Recall GATSM outline

EPIR component comes from this part of GATSM:

- **state variables under physical measure \mathbb{P} :**

$$dx(t) = \kappa [x(t) - \theta]dt + \sigma dW(t)$$

- **short rate:** $r(t) = a_0 + b'_0 x(t)$

BYRP comes from this part of GATSM:

- **Specify risk adjustment:** $\Pi(t) = \sigma^{-1} [\gamma + \Gamma x(t)]$

Combining the two parts above gives the yield curve:

- **state variables under risk-adjusted measure \mathbb{Q} :**

$$dx(t) = \tilde{\kappa} [x(t) - \tilde{\theta}]dt + \sigma d\tilde{W}(t)$$

- **forward rates:** $f(t, \tau) = b'_0 \exp(-\tilde{\kappa}\tau) x(t) + \text{VE}(\tau)$
- **interest rates:** $R(t, \tau) = \frac{1}{\tau} \int_0^\tau f(t, u)du$

**All within shadow/LB framework
to account for UMP/lower-bound periods**

From shadow/LB model estimation

R data, observed yield curve, is composed of:

- **EPIR**: Expected Policy Interest Rate component
- **BYRP**: Bond Yield Risk Premium component
- negligible residuals, i.e. **R model** \simeq **R data**, so ...

$$\mathbf{R\ model/data} = \mathbf{EPIR} + \mathbf{BYRP}$$

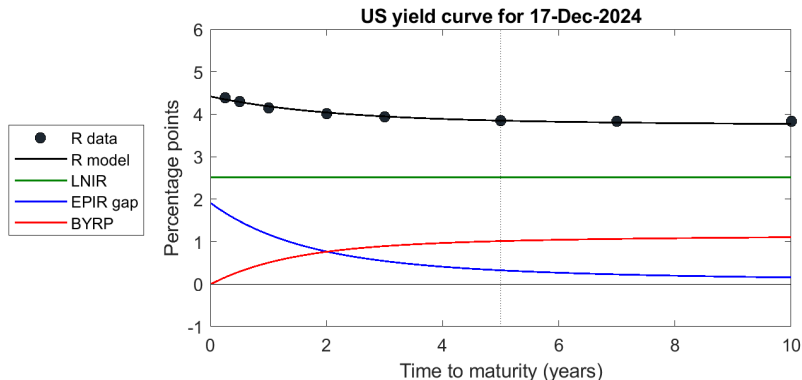
Estimation includes survey data specific to EPIR:

- expectations of 3-month & 10-year rate for range of horizons
- expectations of long-horizon GDP growth and inflation
 - long-horizon nominal GDP growth is a proxy for steady state PIR, i.e. nominal natural interest rate
 - **LNIR**: Long-horizon Natural Interest Rate (nominal)

Useful to use **EPIR gap** = **EPIR** - **LNIR**, so:

$$\mathbf{R\ model/data} = \mathbf{LNIR} + \mathbf{EPIR\ gap} + \mathbf{BYRP}$$

Example: recent yield curve decomposition for US



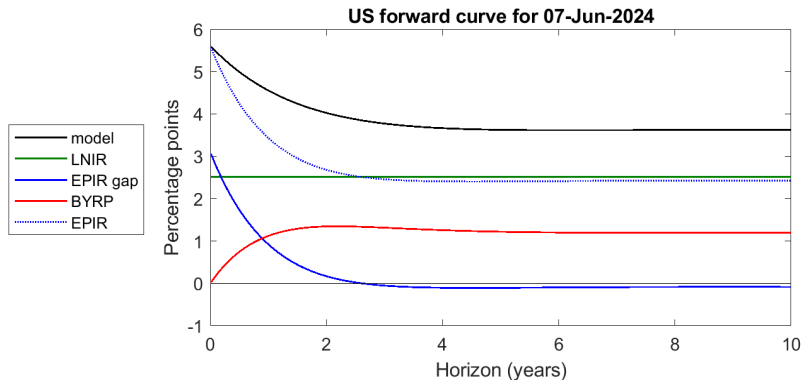
yield curve view gives average of paths to each maturity

i.e. average of expected EPIR path & EPIR gap

forward rate view gives the path themselves

expected path of EPIR & EPIR gap (and marginal BYRP)

Forward rate view for the decomposition



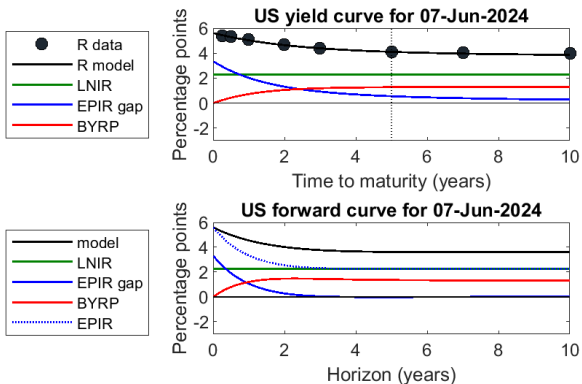
EPIR > LNIR: +ve EPIR gap, with expected decline

⇒ restrictive monetary policy stance, with gradual easing

+ve risk premium component

consistent with inflation concerns and Quantitative Tightening

Both figures on the same page ...



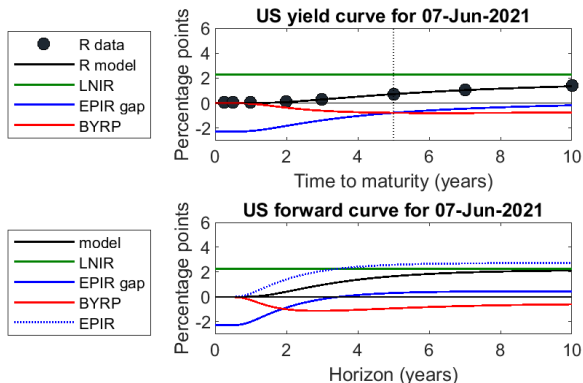
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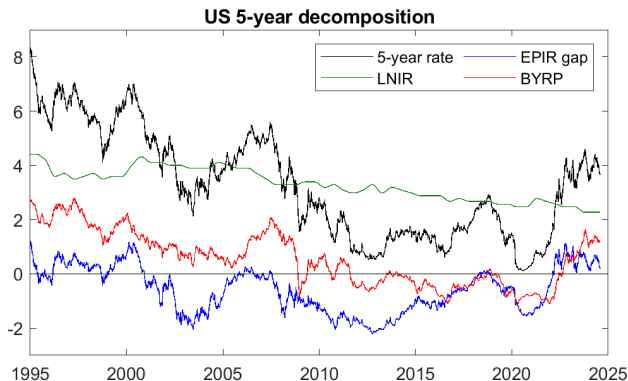
consistent with inflation concerns and Quantitative Tightening

... for comparing to three years earlier



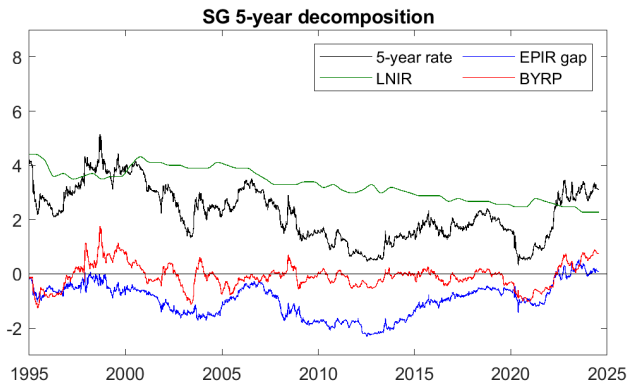
EPIR < LNIR: -ve EPIR gap, with expected increase
 ⇒ stimulatory monetary policy stance, with eventual tightening
-ve risk premium component
 consistent with growth concerns and Quantitative Easing

Time series of 5-year yield decomposition for US



- LNIR**: gradual decline (lower GDP growth and π expectations)
- EPIR gap**: mainly easy monetary policy (\sim 2003, GFC, COVID)
- risk premium**: some large declines (lower π concerns, safety)
most -ve during COVID period, then material reversal

Time series of 5-year yield decomposition for Singapore

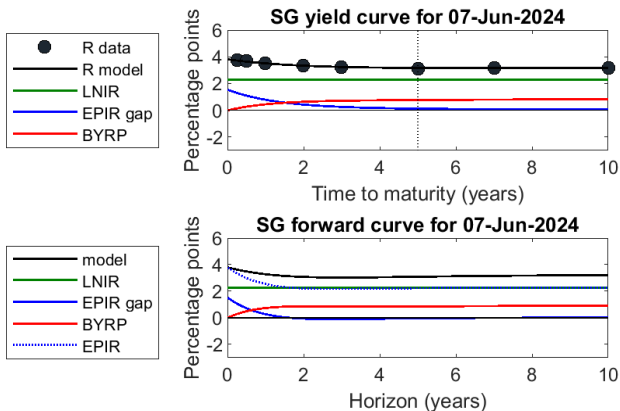


LNIR: I have used the US LNIR (and model parameters)

“EPIR gap”: should reflect UIP & $\mathbb{E}[\text{path } \Delta\text{USD}/\text{SGD}]$

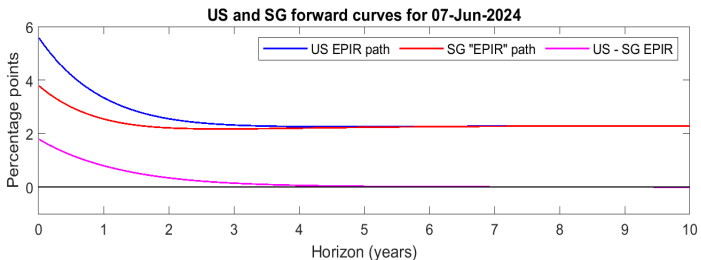
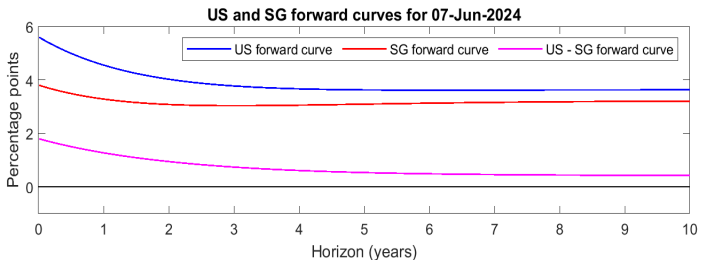
risk premium: quite steady, unlike other economies
(some similarities to US since ~ 2008)

Example: Singapore yield curve & forward rate views



Profiles all similar to US 7-Jun-24 example
IN PRINCIPLE should reflect **US yield curve views**
less $\mathbb{E}[\text{path } \Delta\text{USD/SGD}]$ on 7-Jun-24

Example: Compare US and SG results for 7 June 2024



Singapore context: recall UIP points from first section

General version of UIP:

- $\mathbb{E}[\text{path } \Delta\text{NEER}] = \mathbb{E}[\text{foreign IR path}] - \mathbb{E}[\text{IR path}]$
- i.e. interest rate path differences imply NEER path

US – SG paths from previous slide are examples of the inferred $\mathbb{E}[\text{path } \Delta\text{USD/SGD}]$ for 7-Jun-24 (note: annualized basis). There are two alternative inferences, i.e. based on:

- forward rates, which include risk premiums
- expected short-maturity rates, without risk premiums

Open questions:

- is either example plausible? which is better?
- what would $\mathbb{E}[\text{path } \Delta\text{USD/SGD}]$ inferences look like at different points in time?
- how would they look for LB/UMP periods?

Would this approach produce sensible and useful results?

5. Conclusion

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Conclusion

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Singapore context

- exchange rate is key
- interest rates matter for transmission & UIP

Term structure models

- use Gaussian models as the shadow term structure
- and overlay with lower-bound framework

Application: shadow rates

- -ve shadow rates indicate Unconventional Monetary Policy
- -ve Singapore results indicate “import” of global UMP

Application: yield decompositions

- term structure models are basis for yield curve decompositions
- potential application to gauging Singapore's expected NEER

Preliminary ideas & results, and many open questions

Will look forward to feedback and discussions