Documentation for Shadow Short Rate Estimates

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Summary

This note documents the Shadow Short Rate (SSR) estimates on my website, ljkmfa.com. In particular, I discuss material revisions to the previous series of SSR estimates for the G4 economies and the aspects underlying those changes.

In summary, the changes I have made are:

- Introducing a time-varying lower bound (LB) to accommodate negative policy interest rate settings.
- Calibrating the volatility of SSR estimates to be similar to the volatilities of policy and 3-month interest rates prior to the 2008 LB environment.
- Including additional yield curve data for the estimation, both a longer time series due to evolution of data since the 2016 re-estimation, and also incorporating further longer-maturity yield curve data.

These changes are required to make the models fit for purpose in the renewed global LB environment, including the potential for negative/more-negative policy interest rates and to allow for the low level of global yield curves.

SSR series are estimated quantities that vary with model specification (particularly the LB value) and the data used for their estimation. The material revisions arose because I have introduced all of the changes noted above at once. The resulting SSR series are summarized in figure 1 along with the previous SSR series.

The new G4 SSR estimates improve on the previous series, as I discuss in the "vetting" subsections for each SSR series within section 4. Notably, the negative SSR estimates are less extreme, and the SSR series generally evolve consistently with conventional and unconventional monetary policy.

Allowing for a time-varying LB has allowed me to add SSR estimates for Switzerland to the suite, whereas previously the large negative policy interest rate setting would be inconsistent with an assumed zero LB. I have also added SSR estimates for Canada, Australia, and New Zealand.

My future SSR updates will include more frequent re-estimations, further refinements, and additional vetting for the SSR series. However, I expect any resulting changes to the SSR series to be small relative to those documented in this note.

No single monetary policy metric can ever be ideal, either SSR estimates or even policy interest rates during times of conventional monetary policy. Hence, I intend to introduce other monetary policy metrics to the suite over time.

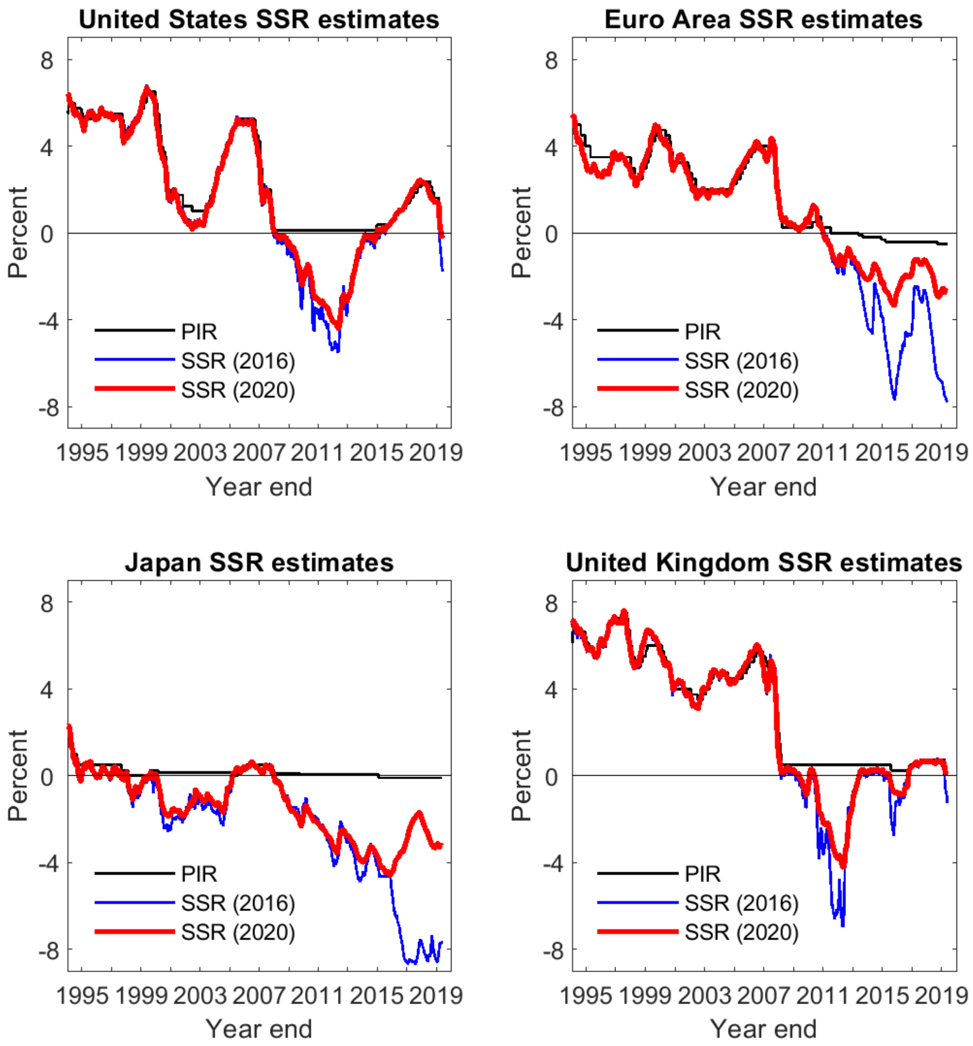

Figure 1: G4 policy interest rates (PIRs) and Shadow Short Rate (SSR) estimates from the 2016 model and the 2020 model.

1 Introduction

In this note, I provide documentation for the Shadow Short Rate (SSR) estimates on my website, ljkmfa.com. It updates the documentation from 2016, which in turn updated the documentation from 2014, available on my former Reserve Bank of New Zealand website.[1]

Section 2 provides a short non-technical overview of the concepts underling the SSR and its estimation, which may be skipped by readers already familiar with the topic. In section 3, I provide an overview of the shadow/lower-bound (LB) model specification and the yield curve data used to produce the new SSR estimates. Further details are contained in appendix A.

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[1] Respectively, see https://www.rbnz.govt.nz/-/media/ReserveBank/Files/Publications/Research/additional-research/leo-krippner/5892888.pdf, and https://www.rbnz.govt.nz/-/media/ReserveBank/Files/Publications/Research/additional-research/leo-krippner/2014Documentation_20140930.pdf.

The main part of this note is section 4, where I discuss the new SSR series for each of the G4 economies in turn. The main objective is assess the consistency of each G4 SSR series with the evolution of conventional and unconventional monetary policy in each economy. This descriptive process provides an initial means of vetting the G4 SSR estimates, whereas my vetting was previously focussed on the US. In the US section, I also summarize updated research that supports using a two-factor shadow/LB model and yield curve data with maturities out to 30 years to estimate SSR series, which is how I obtain all of my SSR series.

2 SSR concepts and estimation

SSR estimates are intended to provide a quantitative indicator for the stance of monetary policy over both conventional monetary policy (CMP) and unconventional monetary policy (UMP) environments. Specifically, SSR estimates are similar to the policy interest rate (PIR) and closely related short-maturity interest rates during CMP periods, but can freely take on negative values during UMP periods to account for UMP measures employed in addition to the near-zero PIR.

As background, central banks have conventionally used their PIR as the primary "tool" for operating monetary policy when it could be freely lowered (or raised). During such CMP times, the PIR or short-maturity interest rates close to the PIR (abbreviated to short rates hereafter) are also typically used as the "metric" for the stance of monetary policy. That is, a lower (or higher) level of the PIR and hence short rates is generally taken to indicate more stimulus (or damping) for inflation and economic activity.

Following the Global Financial Crisis (GFC, which is often referred to as the Great Recession in the US), central banks of all G4 economies entered a UMP environment where, in addition to the near-zero PIR, UMP tools such as more-explicit forward guidance, credit easing, and quantitative easing (QE) have been employed. During such UMP times, the PIR or short rates no longer provide an adequate metric for the stance of monetary policy, because the additional UMP tools employed result in an overall monetary policy stance that is more stimulatory than suggested by near-zero PIRs or short rates alone.

One proposal for quantifying the overall stance of monetary policy in UMP times is to use SSR estimates. I first suggested this in the papers Krippner (2012a,b, 2013), and the idea has gained further attention since then.[2] The main principles underlying SSR estimates are:

- Yield curve data, i.e. market-quoted interest rates of different times to maturity, are influenced by the PIR and UMP tools (when the latter are in use).

- The yield curve may be considered as two components:

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[2] Notable examples are Bullard (2012, 2013) in the context of United States monetary policy, and a reference in Draghi (2019) in the European context, although most uses are for research by analysts, central banks, and academics. My shadow/lower-bound yield curve model was first developed in the paper Krippner (2011), and a more definitive treatment is the book Krippner (2015b). Wu and Xia (2016) produces SSR estimates from a model analogous to mine, and those estimates are also widely used.

  - a "shadow yield curve" without a near-zero LB constraint that can therefore freely take on negative values; and
  - a "physical currency option effect" that results in a near-zero LB constraint on interest rates.[3]

- Applying a "shadow/LB" model to yield curve data allows the shadow yield curve and the option effect components to be separately estimated.

- The SSR is then the shortest-maturity interest rate on the shadow yield curve, just like the PIR is the shortest-maturity interest rate on the actual yield curve.

These concepts are readily illustrated with the stylized examples in figure 2:

- In the UMP example, which is akin to the United States in 2011, the near-zero PIR plus UMP tools have led much of the yield curve data to sit close the near-zero LB constraint. Hence the option effect is high and the shadow yield curve has many very negative values. The resulting SSR estimate is –5%.

- In the CMP example, the positive PIR without UMP tools means the yield curve data is essentially unconstrained. Hence the option effect is negligible and the shadow yield curve almost coincides with the actual yield curve. The result is a positive estimated SSR that pretty much equals the PIR of 1% in this example.

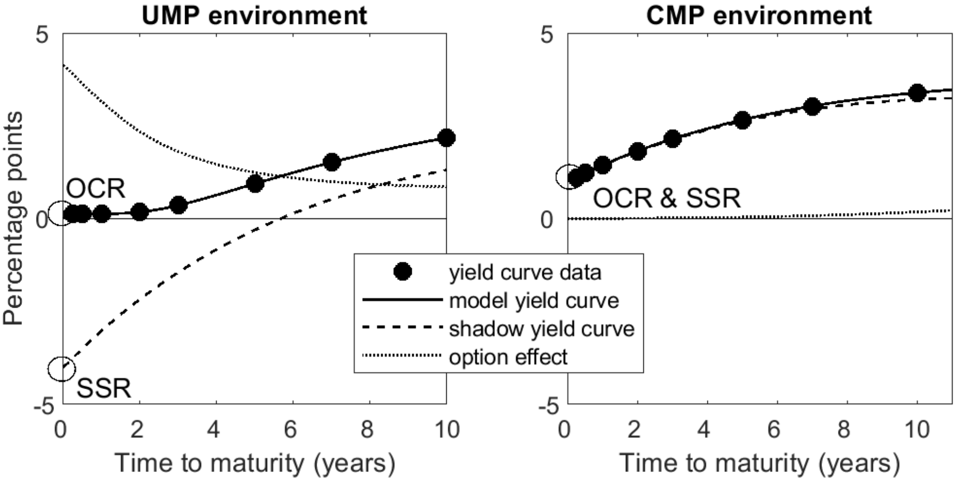

Figure 2: Stylized examples of yield curve data in UMP and CMP environments, and the SSR estimates obtained by applying my shadow/LB model.

Because the shadow/LB model is estimated from yield curve data across UMP and CMP periods, the resulting SSR series provides a consistent and comparable metric for the stance of monetary policy over both periods. Therefore, the levels and changes of SSR estimates in the UMP period may be broadly interpreted in a similar manner to the levels and changes of the PIR or short rate in the CMP period. However, there are several reasons why SSR estimates should not be treated as literally as the PIR or short rates:

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[3] Physical currency essentially offers an interest rate of zero percent. Hence, in the absence of changes to the physical currency framework, there is a limit to how negative the PIR could be set; below a so-called "effective lower bound", people and/or institutions would begin to consider the cost/benefit of the option to hold physical currency instead of account balances subject to a negative interest rate.

- The SSR is an estimated value rather than a setting like the PIR or short rate quoted in markets. Hence, any SSR series will (unavoidably) vary with the model specification and data used for its estimation. For that reason, rather than presuming that an estimated SSR will be useful as a metric for monetary policy in UMP times, it is important to vet any SSR series. The individual subsections in section 4 provide descriptive vetting for each SSR series with respect to CMP and UMP, and additional perspectives for the US. But even once SSR series are vetted, the magnitudes of negative SSR estimations can easily vary by fractions of a percentage point on re-estimations, and sometimes more for UMP periods if the lower-bound setting in the model needs revisiting in light of central bank communications and/or actions.

- The SSR is not a market rate at which borrowers and lenders can transact, particularly in UMP times when the PIR and short rates will remain close to zero while the SSR may become increasingly negative. Hence, SSR declines in UMP times will not result in the same cashflow effects from interest payments and receipts as PIR cuts in CMP times. From that perspective at least, the SSR transmission to the economy may differ from the PIR transmission in CMP times. In other words, the levels and changes in SSR estimates when they are negative should not necessarily be expected to influence the macroeconomy in the same way as PIR or short rate levels and changes in CMP periods.

Notwithstanding the caveats above, the point remains that an SSR series provides a better metric for the overall stance of monetary policy in UMP periods than the near-zero PIR or short rate alone. The SSR series also retains the familiar basis of a single interest rate, as opposed to considering the transmission effects of the PIR and each UMP tools separately. A particularly useful corollary in that regard is that existing models with short-maturity interest rates as an input (such as currency or macroeconomic forecasting models) can still be used, but with the SSR series substituted for the short-maturity interest rate series (or the SSR series could be used just for the UMP period, given the PIR and SSR are similar during CMP periods). Indeed, such substitutions are one of the main uses of SSR estimates for research purposes, and the discussion in section 4.1.1 provides examples of their successful use in macroeconomic models.

3 Summary of model estimation and changes

The essential aspects of the shadow/LB model and the estimation used to produce the SSR estimates are unchanged from my 2016 documentation. That is:

- The model I use to estimate all SSR series is the two-factor arbitrage-free Nelson-Siegel model (i.e. with Level and Slope state variables/factors) within the Krippner (2015) shadow/LB framework. For brevity, I hereafter denote the combined model and framework as the K-ANSM(2).

- The yield curve data I use to estimate the models are daily with maturities out to 30 years.

In section 4.1.1, I provide a detailed discussion on why a two-factor shadow/LB model estimated with data out to 30 years is preferable to alternative choices of specifications and/or data sets.

The changes to the model and its estimation are:

- I have specified the K-ANSM(2) to allow for a time-varying LB, and I have calibrated the volatility of the SSR estimates to be similar to the volatilities of PIR and 3-month interest rates prior to the 2008 LB environment. I provide more details on these aspects shortly below. In the previous model, I used a fixed LB of 12.5 bps (basis points, where 1 bp = 0.01 percentage points) for all economies, essentially adopting the US value, and the SSR volatility was unconstrained. However, a fixed LB of 12.5 bps has become obviously inappropriate for the Euro Area and Japan, given the material and persistent negative PIR settings in those economies.[4] It would also be inappropriate for economies like Switzerland, given a –75 bp PIR setting has been in place since January 2015.
- I have used more yield curve data in the estimation of the models. The main change is to include more maturities. i.e. I have added the 4-, 15-, and 20-year interest rates, so the list of maturities is now: 0.25, 0.5, 1, 2, 3, 4, 5, 7, 10, 15, 20, and 30 years. The additional maturities provide further information on the shape of the yield curve, which is desirable in the prevailing environment where much of the yield curve data for all G4 economies is relatively close to the LB. Previously, I used the maturities of 0.25, 0.5, 1, 2, 3, 5, 7, 10, and 30 year because they are the standard benchmark maturities for the US bond market. The second data change relative to the 2016 estimation is simply that the passage of time has added several more years of yield curve data to the estimations.

Regarding the time-varying LB in the new models, I now use a cumulative minimum (CumMin) of the actual PIR, subject to a maximum of 12.5 bps, i.e.:

$$\mathrm{r}_{\mathrm{LB},t} = \min\left\{12.5, [\text{actual PIR setting}]_t\right\} \tag{1}$$

The value of 12.5 bps is the mid-point of the US 0-25 bp range for the Federal Funds Target Rate (FFTR) set by the Federal Reserve on 8 December 2008. I use the US as a precedent for the market-perceived LB for all of the G4 economies prior to any lower PIR setting, given the US is the world's largest economy and it was also the first major economy to set its policy interest rate at a LB value following the GFC. For other economies, the 12.5 bps value is representative of the ex-ante ambiguity for markets on whether other G4 central banks with different institutional arrangements for operating their PIR mechanism might adopt a PIR setting of 0 bps or 25 bps.

Obviously the PIR for some central banks has moved to settings lower than 12.5 bps following the GFC (or the COVID-19 crisis for the UK), and even to negative PIR settings

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[4] I considered using a time-varying LB when I last changed the models in 2016, but decided against it for three reasons: (1) the magnitude of negative PIR settings were relatively small at the time; (2) the exemptions for banks around negative PIR settings, e.g. in Switzerland and Japan (and now the Euro Area), left some ambiguity about the effective PIR; and (3) it was uncertain if negative PIR settings would be persistent.

for the Euro Area and Japan. If/when a central bank makes settings lower than 12.5 bps, the CumMin mechanism moves the LB to the new PIR setting from that point onward. Likewise, the LB will continue to step down if/when subsequent lower PIR settings are made.

The CumMin mechanism is one of the five options investigated in Kortela (2015) for the Euro Area and it is consistent with the behavior of yield curve data in practice. That is, the yields for all maturities tend to respect the prevailing PIR or the short-horizon anticipation of a forthcoming cut to the PIR. Even when the market occasionally anticipates a larger cut to the PIR than is subsequently delivered by the central bank, all of the yield curve data then reverts to respecting the prevailing PIR setting. These empirics, particularly for the longer-maturity yield curve data, are consistent with the market treating the current PIR setting as the LB that is expected to apply for long horizons into the future.[5]

Table 1 summarizes the LB settings resulting from the CumMin mechanism for the G4 economies, and also the other four economies for which I have introduced monthly updates of SSR estimations. Note that the LB of 12.5 bps applies across the whole sample for the US, which is consistent with the Federal Reserve never setting a target range for the FFTR lower than 0-25 bps and also never providing an indication (to date) that it might be prepared to adopt such a setting.[6] Note also that I treat the two LB periods in Japan distinctly. That is, after the PIR lift-off in July 2006, I return the LB to the assumed 12.5 bps rather than leaving it at the previous CumMin value of zero.[7] This step-up of the LB to 12.5 bps reflects the intention of the Bank of Japan prior to and initially during second LB period to maintain a small positive PIR; see Ueda (2011).

Calibrating the SSR volatility to be similar to the volatilities of the PIR and 3-month interest rates prior to the 2008 LB environment is less of a fundamental than a cosmetic change. It leads the resulting SSR estimates to be less subject to short-lived movements in the yield curve data and therefore to evolve more smoothly over time, like a PIR setting. In particular, the SSR volatility calibration counters occasional short-lived SSR down-spikes associated with market over-anticipation of forthcoming monetary policy actions that are followed by up-spikes as the market adjusts to the realized monetary policy settings.

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[5] If the market genuinely expected the LB for longer future horizons to be less than the prevailing PIR, then one would expect to see the short-maturity yield curve data respect the prevailing PIR, while longer-maturity data would reflect a lower future-anticipated LB.

[6] An internal memo dated 5 August 2010, Burke, Hilton, Judson, Lewis, and Skeie (2010), later made public on 29 January 2016, showed that the Federal Reserve considered negative PIR settings following the GFC, but obviously decided against them at the time. More recently, in response to a question at the 18 September 2019 FOMC press conference, FOMC Chairman Powell made the comment: "I do not think we'd be looking at using negative rates. I just don't think those will be at the top of our list."; see Powell (2019).

[7] Of course, PIR settings ultimately again fell below 12.5 bps in the second LB period, and the CumMin mechanism then sets the LB to those PIR settings as they are realized. Also note that I have tested the alternative of applying the CumMin mechanism across the entire sample, and the resulting SSR estimates are not greatly different from those obtained with the two distinct LB periods.

Table 1: LB settings for the K-ANSM(2) estimations

Economy	LB	Date applied from	Notes (1,2)
United States	12.5	All dates	12.5 bps
Euro Area	12.5	<5-Jul-2012	12.5 bps
	0	5-Jul-2012	PIR setting
	-10	5-Jun-2014	PIR setting
	-20	4-Sep-2014	PIR setting
	-30	3-Dec-2015	PIR setting
	-40	10-Mar-2016	PIR setting
	-50	12-Sep-2019	PIR setting
Japan	12.5	<25-Feb-1999	12.5 bps
	0	25-Feb-1999	PIR setting
	12.5	14-Jul-2006	12.5 bps
	10	19-Dec-2008	12.5 bps
	5	5-Oct-2010	PIR setting
	-10	29-Jan-2016	PIR
United Kingdom	12.5	<19-Mar-2020	12.5 bps
	10	19-Mar-2020	PIR setting
Switzerland	12.5	<18-Dec-2014	12.5 bps
	-25	18-Dec-2014	PIR setting
	-75	16-Jan-2015	PIR setting
Canada	25		12.5 bps
Australia	25		12.5 bps
New Zealand	25		12.5 bps

Notes: (1) The 12.5 bp value is the precedent obtained from the US, as explained in the text. (2) The PIR settings referred to in this table for the Euro Area are the ECB deposit rates, which became the most relevant to the yield curve data after the onset of full allotment announced on 8 October 2008. The PIR settings for Japan are the target for the overnight call rate prior to 5 October 2010, the mid-point of the 0-0.1% target range for the overnight call rate from 5 October 2010 prior to 29 January 2016, and the deposit rate from 29 January 2016. The PIR setting for the UK is the Bank Rate.

Figure 3 illustrates the benefit of applying SSR volatility calibration in the case of the Euro Area. While there are several examples in figure 3, the period around the 12 September 2019 ECB meeting provides the clearest example of markets overanticipating the PIR cut, UMP actions, and/or the potential for future monetary policy easing. That is, panel 2 of figure 3 shows that the yield curve data fell well below the current and subsequently realized PIR setting (of –50 bps), e.g. the 1- and 5-year OIS rates reached respective lows of –63 and –71 bps prior to the meeting, and subsequently rose above –50 bps.

The SSR estimates obtained without the volatility calibration (blue) reflect the yield curve movements as a large fall in the SSR, followed by a large rise, which is obviously counter to the typical properties of PIR in CMP times. The new SSR estimates obtained with the volatility calibration (red) eliminate most of the fluctuations, and hence behave more like a PIR in CMP times.

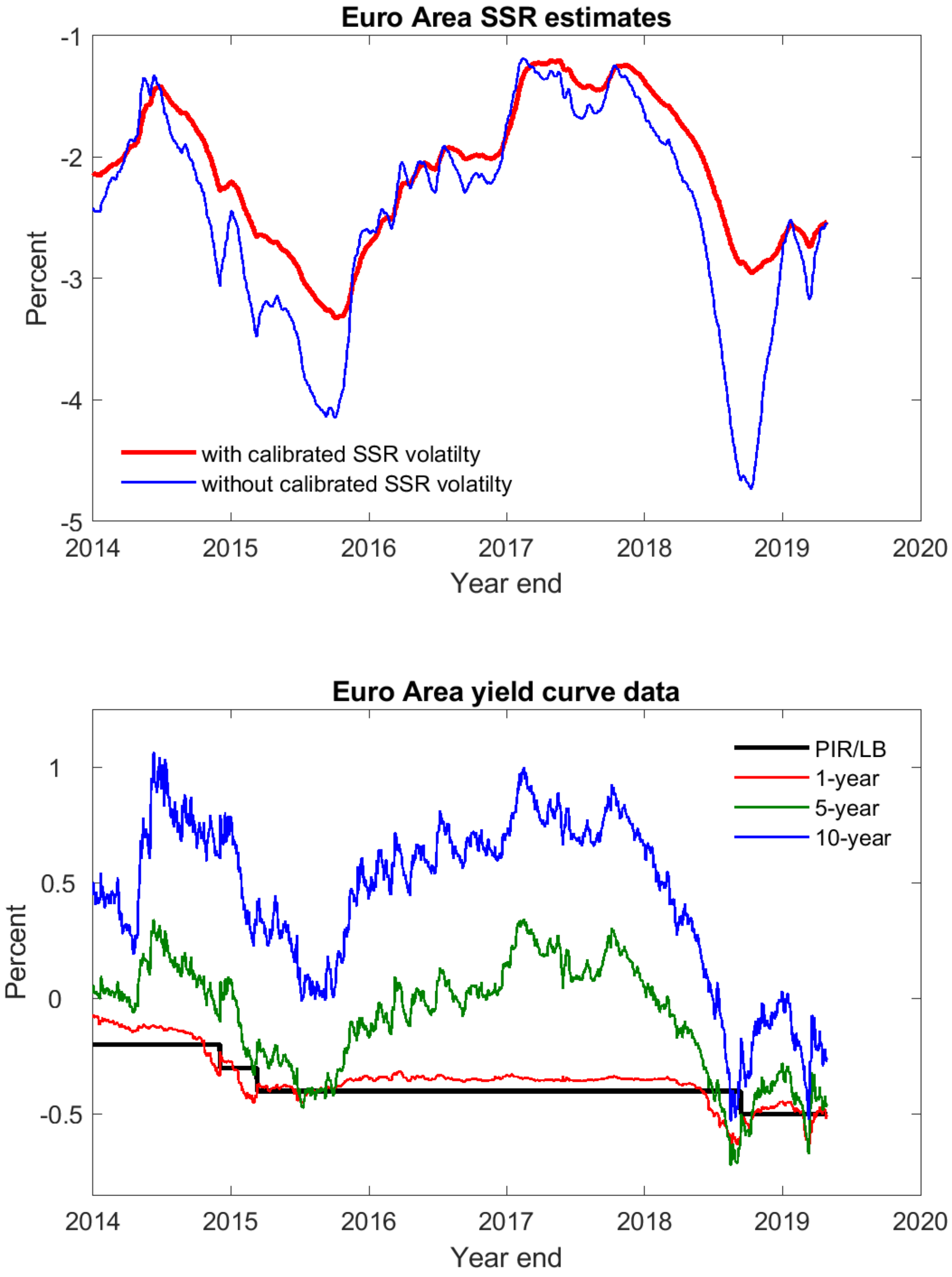

Figure 3: Example for the Euro Area that illustrates the benefits of volatility calibration for SSR estimation. Without such calibration, the SSR estimates are subject to short-lived spikes.

4 Discussion on G4 SSR estimates

In this section, I present and discuss each of the new G4 SSR estimates in turn. For each of the G4 economies, I have produced a figure containing:

- the PIR (black);
- the 30-year interest rate (green);
- the new estimated SSR series (red);
- the previous estimated SSR series (blue).

Each figure includes a subplot for the full sample period and a subplot for the LB period (from end-September 2008 onward) when the SSR estimates are predominantly negative. The latter period therefore allows a visual assessment of whether the evolution of the SSR is consistent with the evolution of major UMP events. I have indicated those events with "down" arrows for UMP easings, and "up" arrows for UMP tightenings.

Aside from comparing the new SSR series to the previous series, the main aim from each figure and the associated discussion is an informal "vetting" of the new SSR series, with a focus on the consistency of each SSR series with the evolution of CMP and UMP, or sometimes the yield curve data.

As background, and as figure 1 has already clearly illustrated even for my preferred approach, SSR estimates can vary greatly with seemingly innocuous yet justifiable choices for the model specification and the data used to estimate them. Hence, while it is relatively straightforward to produce SSR estimates, it should not be presumed that any SSR series will necessarily be useful as a quantitative metric. The only way to determine whether an SSR series is useful or not is through a vetting process, or in other words, "trust, but verify".[8]

For the US, in addition to the informal vetting from the visual assessment, I also summarize formal work that establishes why I choose to estimate all SSR series using a two-factor shadow/LB model estimated with data out to 30 years. These perspectives provide additional vetting for the US SSR estimates, and I intend apply these perspectives to all of the SSR series going forward. But even for this note, it's worthwhile mentioning that the formal US results essentially quantify what is already apparent from the visual assessment in the figures.

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[8] For completeness, an alternative, recommended in Bauer and Rudebusch (2016) and Christensen and Rudebusch (2015), after illustrating the variation in SSR estimates, is not to use them as a monetary policy metric at all. I think this recommendation is premature, given that SSR series with appropriate vetting have been shown to be useful, even if not perfect (e.g. see the macroeconomic discussion in section 4.1.1), and no clearly superior alternative metric/s have yet been developed (to my knowledge). Hence, I prefer to retain SSR estimates, even while considering other monetary policy metrics to include in a wider suite.

4.1 United States

Figure 4 shows the new US SSR series (red), which has been revised to be less negative than the previous series (blue). For both the old and the new SSR series, the magnitudes of the negative SSR estimates during the easing/tightening cycles in the UMP periods are similar to the magnitudes of the PIR and SSR estimates during the CMP period.

The profile of the new SSR estimates also remain as previously, and are consistent with the unconventional monetary policy (UMP) actions undertaken by the Federal Reserve. That is:

- The SSR remains close to the FFTR during the CMP period prior to the onset of the UMP period November/December 2008. Note that the deviations of the SSR from the FFTR are due to the former being estimates from yield curve data, and hence the SSR estimates will reflect aspects such as the anticipation of PIR changes and/or forward guidance on the FFTR. For example, the deviations were most notable around 2003 when, in response to the deflation scare at the time, the Federal Reserve provided more explicit forward guidance on the PIR than usual.

- The SSR generally trended down from the onset of the LB period in November/December 2008 (QE1 was announced on 25 November 2008, and the 0-0.25% FFTR range was set on 16 December 2008) to the first indication of tightening in May 2013 (as discussed in the following bullet point). The notable exception to the downward trend over this period was the SSR increase from November 2010 to April 2011, which I have shaded in panel 2 of figure 3. This period corresponded to a change in market expectations for an early PIR lift-off, e.g. the 1-year/1-year forward OIS rate rose above 0.50% in November 2010 to as high as 1.31% in April 2011, which was in turn due to the relatively positive economic data at the time (and perhaps reinforced by the introduction of QE2 on 3 November 2010, which had been foreshadowed on 27 August 2010 by FOMC Chair Bernanke at Jackson Hole).

- The SSR generally trended up from the first indication of QE3 tapering, via the comments by FOMC Chair Bernanke on 13 May 2013 that lead to the so-called "taper tantrum" (I have shaded this episode in panel 2 of figure 3), throughout the actual tapering itself (beginning on 18 December 2013 and ending on 29 October 2014), until the "lift-off" on 16 December 2015 when the FFTR range was raised to the 0.25-0.50% range. The notable exception during this period was the SSR decrease between the indication and onset of tapering. This occurred as members of the FOMC tried to reassure markets that the onset of tapering should not be interpreted as synonymous with an imminent PIR lift-off.

- The SSR remained close to the FFTR during the FFTR tightening/easing cycle, again with the SSR estimates anticipating the realized FFTR increases and decreases.

- The SSR has again returned to negative levels following the 15 March 2020 announcement of the FFTR cut to the 0-0.25% range, accompanied by a $700 billion QE programme and a commitment to ensure smooth market functioning.[9]

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[9] The latest update in this regard was from the 29 April 2020 FOMC meeting, see https://www.federalreserve.gov/monetarypolicy/files/monetary20200429a1.pdf.

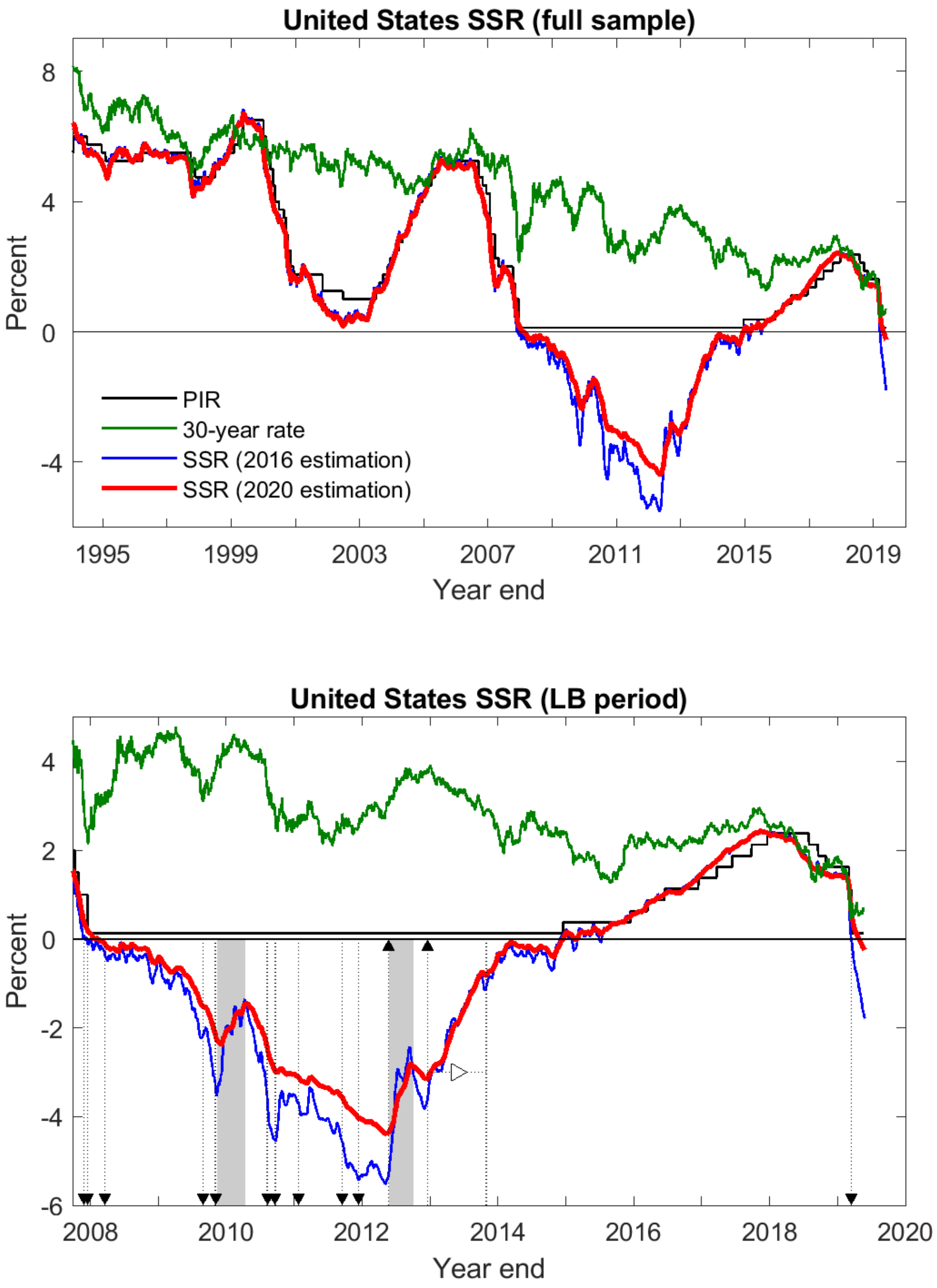

Figure 4: Panel 1 contains the old and new US SSR estimates for the whole sample, along with the PIR and the 30-year interest rate. Panel 2 contains the same series in panel 1 but from September 2008, and also "down" and "up" arrows to respectively indicate major UMP easing and tightening events.

4.1.1 The case for two-factor/30-year data SSR estimates

My preference for estimating SSR series from a two-factor shadow/LB model, like the K-ANSM(2), using yield curve data out to maturities of 30 years is essentially because the properties of the resulting SSR series as a monetary policy metric have been shown to be better than the alternatives. The case is most established for SSR estimates for the US, and so the main results from that research are summarized in this additional subsection here.

The main alternative choices to my preferred approach for estimating SSR series is to use a three-factor shadow/LB models and/or yield curve data out to maturities of 10 years. That combination, along with a fixed LB of 25 bps, is the basis used in Wu and Xia (2016, hereafter WX) to produce the WX SSR estimates for the US. However, such models typically produce SSR estimates with counterintuitive properties, from at least the following perspectives:

1. larger sensitivity with minor model and data changes, including marked profile changes and occasional positive values during LB/UMP periods;
2. counter-correlations with major UMP events at a monthly frequency;
3. counter-correlations with financial market data at daily frequencies; and
4. inferior performance when used within macroeconomic models.

Regarding point 1, Krippner (2015a,b, 2020) and Claus, Claus, and Krippner (2018) illustrate the wide variations of the magnitudes and profiles of WX and K-ANSM three-factor SSR estimates with respect to the LB value used and whether yield curve data out to 10 or 30 years is used for estimation.[10] These papers also show that materially positive SSR estimates sometimes occur during the 2010 to 2013 period when the US was still in the post-GFC LB/UMP environment. Using the K-ANSM(2) produces SSR estimates with similar profiles, smaller variations in magnitudes, and no instances of positive SSR values during the entire LB/UMP period.[11]

Regarding point 2, Krippner (2015a, 2020) formalize the visual assessment of major UMP events with SSR estimates by calculating the correlations of SSR changes during the months of each announcement with the easing/tightening indicators for those months. Changes in SSR estimates obtained from the WX model with yield curve data to 10 years are either insignificantly correlated or significantly negatively correlated with the UMP events. In other words, those WX SSR estimates generally rise (fall) on easing (tightening) UMP events, which is counter to the usual response of the PIR or short rates in CMP periods. SSR estimates obtained using the K-ANSM(2) model with yield curve data out to 10 or 30 years, or the WX model with yield curve data out to 30 years (which in this particular case obtains an SSR series similar to the K-ANSM(2) model with yield curve

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[10] WX, Bauer and Rudebusch (2016), and Christensen and Rudebusch (2015) are other examples generally illustrating the variation of SSR estimates with respect to the LB value, the number of factors, and the data used for estimation.

[11] Krippner (2015a) also shows the high correlation of negative SSR levels with lift-off metrics. Such correlation could also be formalized as a vetting test for SSR estimates.

data out to 30 years), have changes that are significantly positively correlated with UMP events.

Regarding point 3, Claus, Claus, and Krippner (2018) test SSR estimates within a framework that quantifies the responses of observed financial market data to monetary policy shocks in the US. Claus, Claus, and Krippner (2018) finds that the responses of SSR estimates from the Krippner three-factor model to tightening UMP shocks are all negative, often significantly, whether yield curve data out to 10 or 30 years are used in those SSR estimations. Using the K-ANSM(2), Claus, Claus, and Krippner (2018) finds that the responses of SSR estimates to tightening UMP shocks are all significantly positive, and the responses are larger and more significant when yield curve data out to 30 years are used to obtain the SSR estimates.

Regarding point 4, Francis, Jackson, and Owyang (2020) uses a small macroeconomic model for the US to test the performance of the SSR series from Krippner (2015b), which are produced from the K-ANSM(2) with data out to maturities of 30 years, and the SSR series from WX, which are produced from a three-factor model with data out to 10 years. Quoting from the introduction of the article: "We also find that the method proposed by Krippner (2015b) delivers stable parameter estimates when the ZLB period is included in the sample. The stability of the VAR parameters lead us to favor the Krippner shadow rate to represent policy during the unique environment at the ZLB. I have undertaken a similar investigation to Francis, Jackson, and Owyang (2020) in Krippner (2020), using SSR estimates from the WX model estimated using data out to 10 years and to 30 years with a variety of LB values. The 30-year dataset produces SSR estimates with a profile and magnitudes closest to my SSR estimates, and using them within a small macroeconomic model results in the least evidence of structural breaks.

One further vetting exercise from a macroeconomic perspective, albeit less formal, is to gauge SSR estimates with a Taylor rule. Hence, figure 4 plots the real-time Taylor (1999) rule rates from Bernanke (2015), my new and old SSR estimates, and the WX SSR estimates.[12] Taylor rule rates are intended to provide a prescription for the PIR based on the macroeconomic inputs of the output gap and inflation, and Bernanke (2015) has used those inputs in real time to ensure they reflect what the Federal Reserve would have had available for monetary policy deliberations at each point in time. The actual PIR often differs from the prescription given the wider set of considerations taken into account when setting monetary policy, but the PIR has nevertheless broadly evolved with the Taylor rule rate during CMP period. Hence, a simple test for an estimated SSR series is to see how it evolves with Taylor rule rate during UMP periods.

On magnitudes, table 2 summarizes the mean values of the Bernanke (2015) real-time Taylor rule rate, a quarterly series, from the March 2009 quarter (the first full quarter within the UMP period) to the March 2015 quarter (the last quarter of the series available in the online data), and then the SSR series over that same period. The means of old and new Krippner SSR series are respectively within 0.2 and 0.4 percentage points of the

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[12] The Taylor (1999) rule is the version often referred to by the Federal Reserve, such as in Bullard (2012, 2013). The WX SSR series was obtained from https://www.frbatlanta.org/cqer/research/wu-xia-shadow-federal-funds-rate.aspx. It was updated to end-month April 2020 at the time of writing, although the Bernanke (2015) data limits the comparisons to March 2015.

mean Taylor rule rate, while the WX SSR series is about one percentage point higher. [13] On profiles, the WX SSR series also takes a counterintuitive step down from mid-2013 to mid-2015, just when the Taylor rule rate begins a steady trend up. Both Krippner SSR series have the upward trend of the Taylor rule rates from mid-2013.

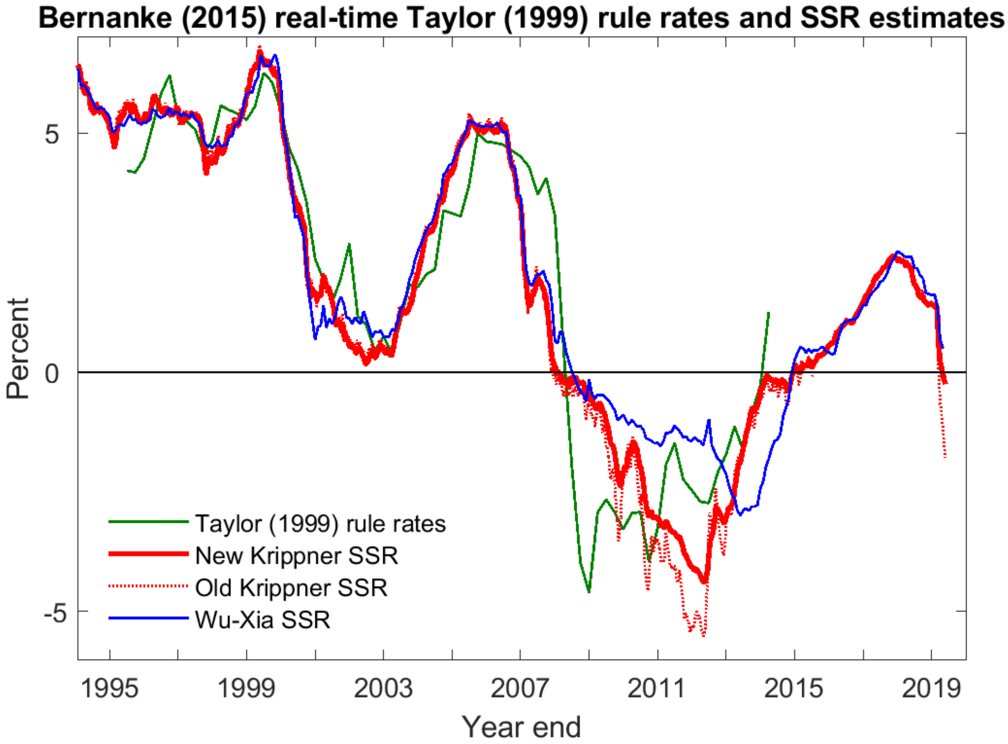

Figure 5: Bernanke (2015) real-time Taylor (1999) rule rates and SSR estimates for the US. The Krippner SSR estimates more closely the magnitude and profile of the rates suggested by the Taylor (1999) rule.

Table 2: UMP mean comparison

	UMP mean
Taylor rule rate	–2.19
Krippner SSR, new	–1.81
Krippner SSR, old	–2.41
Wu-Xia SSR	–1.27

4.2 Euro area

Figure 5 plots the new Euro Area SSR series (red), which has been revised to be much less negative than the previous series (blue). The less negative SSR estimates is a positive feature, in that the magnitudes of the negative SSR estimates during the easing/tightening cycles in the UMP periods are now more similar to the magnitudes of the PIR and SSR estimates during the CMP period. The lows of the previous SSR estimates, at up to twice the magnitude of the PIR and positive SSR estimates were arguably too extreme.

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[13] The SSR estimates from WX are obtained using a LB of 25 bps, which Krippner (2015a, 2020) shows is inconsistent with the yield curve data. Using a smaller LB produces less negative SSR estimates, again see Krippner (2015a, 2020), so the deviation from the Taylor rule would be larger again.

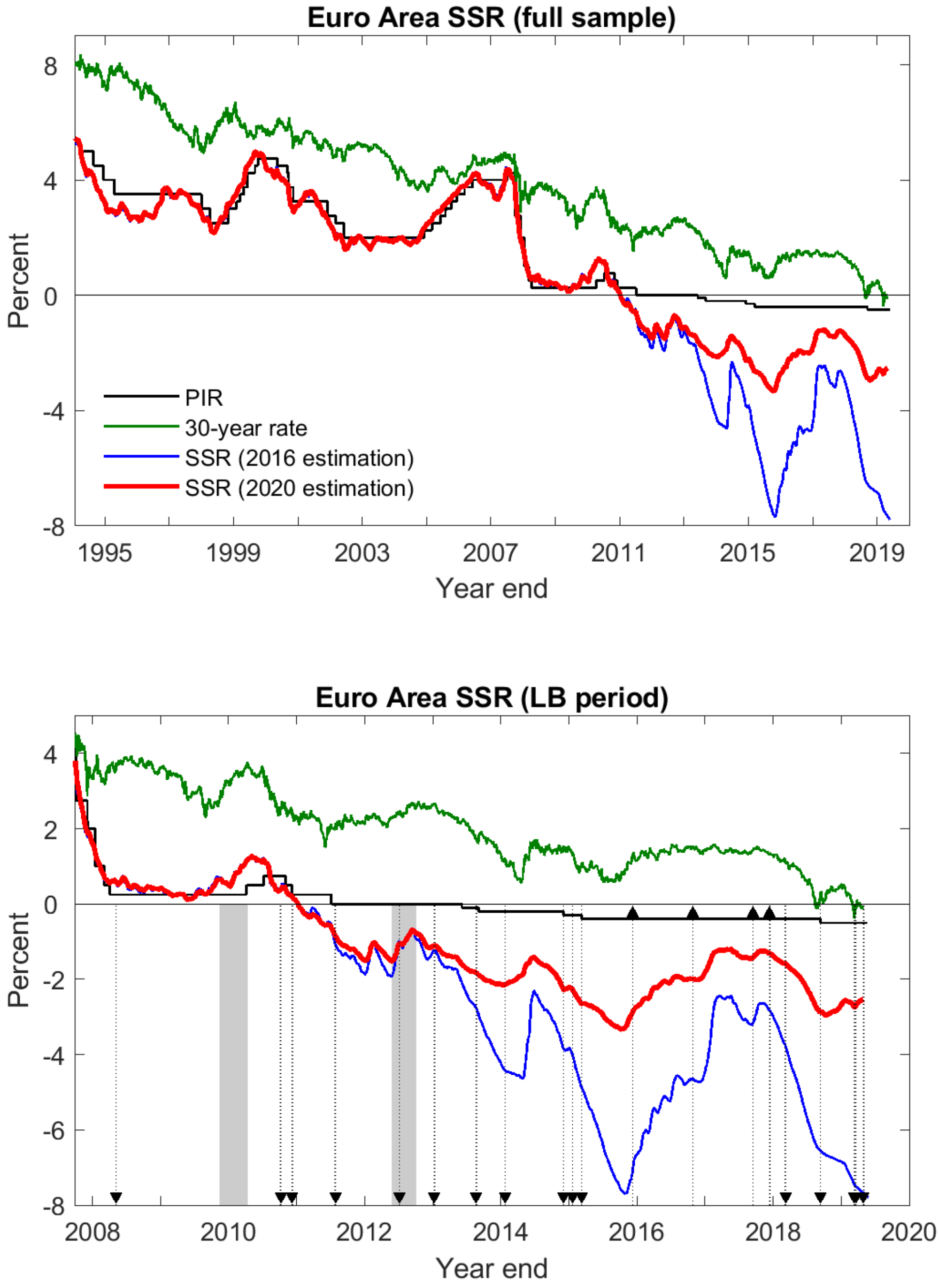

Figure 6: Panel 1 contains the old and new Euro Area SSR estimates for the whole sample, along with the PIR and the 30-year interest rate. Panel 2 contains the same series in panel 1 but from September 2008, and also "down" and "up" arrows to respectively indicate major UMP easing and tightening events.

Despite the large changes in magnitudes, the profiles of the SSR estimates remain as previously, and are generally consistent with the CMP and UMP actions undertaken by the European Central Bank (ECB). That is:

- The SSR remained close to the PIR prior to and during the first easing/tightening phase from 2008 to 2011.[14] The only major UMP event in this period was the simultaneous announcement of long-term refinancing operations and asset-backed purchase program (LTRO2 and CBPP1) on 7 May 2009.[15]

- The SSR generally trended down from the onset of the second easing phase that included more frequent and larger UMP events (e.g. LTROs, explicit forward guidance, ECB President Draghi's "whatever it takes" speech, and asset purchases). The first exception to the downward trend was the US "taper tantrum" episode, as shaded and previously discussed in section 4.1. The second exception is in the first half of 2015, which occurred amidst sharp increases in global longer-maturity yields following their falls to then record lows in April 2015.[16] These exceptions illustrate that global events, via their effect on the domestic yield curve, can influence SSR estimates counter to the domestic central bank's intended stance of monetary policy.

- The SSR generally trended up during the second tightening phase, as the ECB progressively announced the tapering of the APP programme (from the initial 8 December 2016 announcement to the end of the programme on 13 December 2018).

- The SSR has generally trended down during the third easing phase, which began on 7 March 2019 with the announcement of LTRO3. More recently, the SSR has turned up mildly, which followed the 12 September 2019 ECB meeting where the PIR was cut to –0.50% and the APP was reinstated at €20 billion per month. As discussed at the end of section 3, this upturn was due to markets overanticipating the easing actually delivered in September 2019. Figure 3 from section 3 shows that markets also overanticipated the easing delivered in March 2020, e.g. 1- and 5-year OIS rates dropped to -68 bps, but subsequently rose above –50 bps when the PIR was left unchanged.

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[14] The PIR is the mid-point of the Bundesbank Discount Rate and Lombard Rate prior to the introduction of the European Monetary System on 1 January 1999 (as a proxy for a Euro Area PIR), the ECB Main Refinancing Operations rate from 1 January 1999 prior to 8 October 2008, and the ECB deposit rate from the announcement of full allottment on 8 October 2008.

[15] I have not indicated the announcement of LTRO1 and the introduction of the Securities Market Program (SMP) as major UMP easing events, because associated ECB communications at the time indicated that they were not intended to alter the prevailing stance of monetary policy. For LTRO1, see the announcement record "22/082007 Supplementary longer-term refinancing operation" in the table https://www.ecb.europa.eu/mopo/implement/omo/html/communication-history.en.html, which includes the comment: "The position of the Governing Council of the ECB on its monetary policy stance was expressed by its President on 2 August 2007." That date was the previous ECB monetary policy meeting, where the PIR was left at 4.00%. For the SMP, see the announcement record https://www.ecb.europa.eu/press/pr/date/2010/html/pr100510.en.html, which includes the comment: "In order to sterilise the impact of the above interventions, specific operations will be conducted to re-absorb the liquidity injected through the Securities Markets Programme. This will ensure that the monetary policy stance will not be affected." Of course, including or excluding these events makes little difference to the informal visual assessment in this note, and it would be worthwhile testing both ways in more formal analysis.

[16] From my reading, the catalyst for the sharp increases wasn't definitively identified at the time, or since, but rising inflation, an improving US economy, and an imminent US lift-off were mentioned in the financial media at the time, e.g. see https://www.bbc.com/news/business-32622830 and https://money.cnn.com/2015/06/07/investing/interest-rates-fed-bonds/index.html.

4.3 Japan

Figure 7 plots the new Japanese SSR series (red), which has been revised to be much less negative over recent years than the previous series (blue). Like for the Euro Area, a positive feature is that now the magnitudes of the negative SSR estimates during the easing/tightening cycles in the UMP periods are more similar to the magnitudes of the PIR and SSR estimates during the CMP period.

Aside from recent years, to be discussed in the final bullet point below, the profiles of the SSR estimates remain as previously, and are generally consistent with the CMP and UMP actions undertaken by the Bank of Japan. That is:

- The SSR remained close to the PIR during the CMP prior to the zero interest rate policy (ZIRP) introduced in February 1999.
- The SSR declined to mildly negative values after the onset of the ZIRP, then rose back above zero when the PIR was raised to 25 bps in August 2000.
- The SSR declined to moderate negative values following the PIR cut to 15 bps in February 2001, and more so with the progressive announcements of quantitative easing from March 2001 until the end of those programmes in March 2006, when the PIR was set to zero.
- The SSR remained close to the PIR during CMP period from July 2006 and prior to the GFC.
- The SSR generally declined steadily from the end of 2008 to the end of 2016, consistent with the cuts to the PIR and an ongoing series of quantitative and qualitative UMP announcements. The three main exceptions to the trend, as already discussed for the Euro Area in section 4.2, are the US influence in 2010 (the first shaded period), the US "taper tantrum" in 2013 (the second shaded period), and the episode of sharply rising global longer-maturity yields in 2015.
- The SSR rises steadily from late 2016 to late 2018, which is obviously inconsistent with the intentions of the BOJ to maintain an accommodative stance of monetary policy. Indeed, the BOJ maintained its balance sheet expansion over this period and also introduced a new UMP policy "Quantitative and Qualitative Monetary Easing with Yield Curve Control" in September 2016, which was intended to keep 10-year interest rates around zero. Again, the rise in the SSR over this period seems due to the influence of global yield curves, which is apparent from the similar profile of the Japanese 30-year interest rate to the other G4 economies. In particular, the SSR series for Japan closely tracks that for the Euro Area.

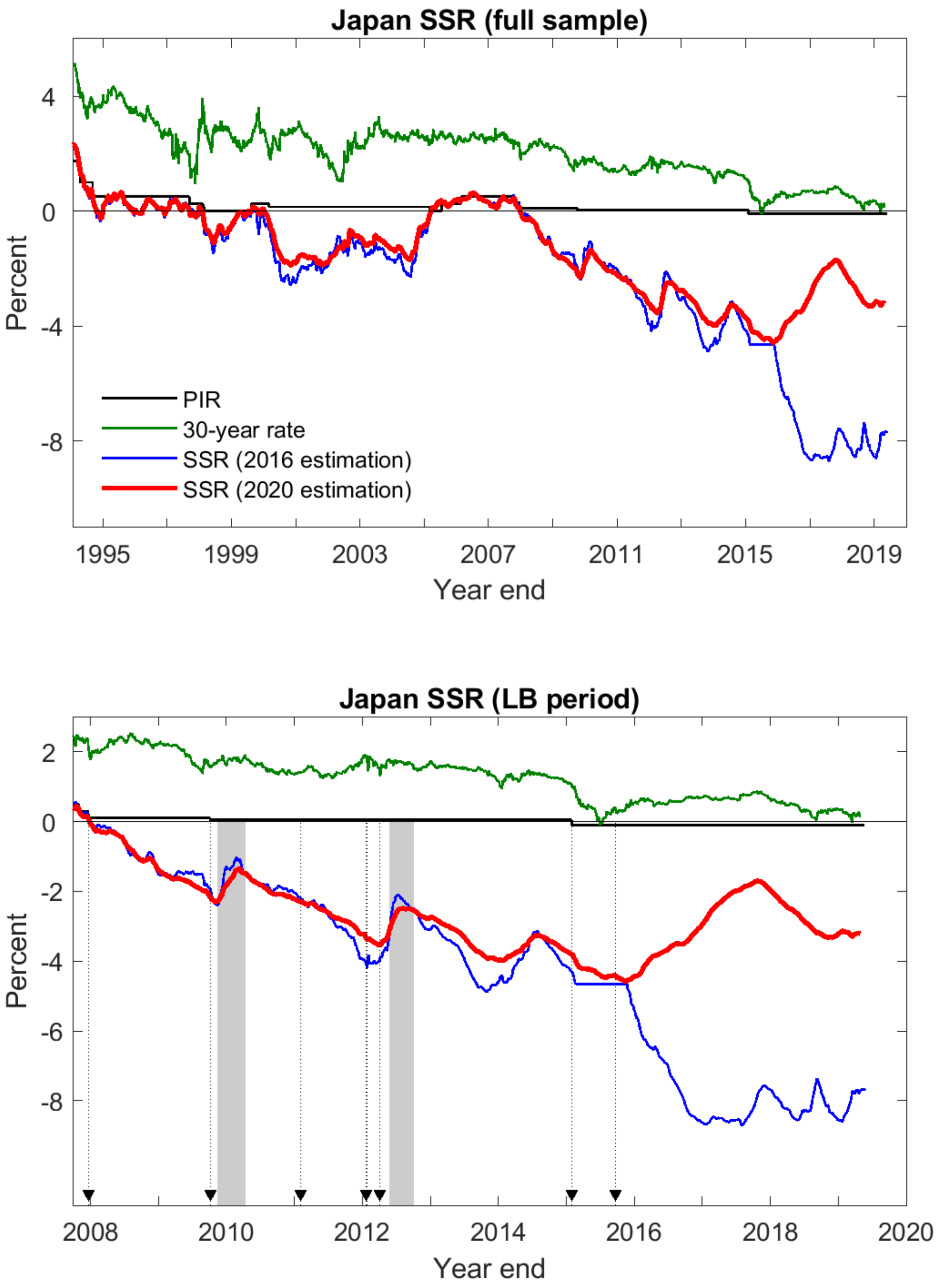

Figure 7: Panel 1 contains the old and new Japanese SSR estimates for the whole sample, along with the PIR and the 30-year interest rate. Panel 2 contains the same series in panel 1 but from September 2008, and also "down" and "up" arrows to respectively indicate major UMP easing and tightening events.

4.4 United Kingdom

Figure 8 plots the new UK SSR series (red), which has been revised to be much less negative than the previous series (blue). As for the Euro Area and Japan, a positive feature is that now the magnitudes of the negative SSR estimates during the easing/tightening cycles in the UMP periods are more similar to the magnitudes of the PIR and SSR estimates during the CMP period.

The profiles of the SSR estimates remain as previously, and are consistent with the CMP and UMP actions undertaken by the Bank of England (BOE). That is:

- The SSR remained close to the PIR during the CMP period prior to the GFC.
- The SSR generally trended down during the following the GFC, and more so as the announced asset purchase amounts were stepped up from 2011. The exception to the downward trend is the US influence in 2010, the first shaded period, as discussed in previous sections.
- The SSR generally trended up from May 2013. The initial part of that increase was coincident with the US "taper tantrum", as shaded and discussed in previous sections, while the increase after August 2013 reflects the combination of BOE's forward guidance conditional on the unemployment rate remaining above 7% (announced 1 August 2013) followed by the unemployment rate approaching that rate later in the year.
- The SSR remained close to but below the PIR from mid-2014 until the Brexit vote on 23 June 2016.
- Following the Brexit vote, the SSR declined in line with the PIR cut from 0.50% to 0.25% and a combination of new UMP actions (the Term Funding Scheme and new asset purchases).
- The SSR increased in line with the non-renewal of the Term Funding Scheme on 3 August 2017 followed by the PIR increase to 0.50% on 2 November 2017, and the SSR remained close to the PIR from this time until the COVID-19 crisis.
- The SSR declined in the wake of the COVID-19 crisis as the PIR was cut and new UMP actions (the second Term Funding Scheme and new asset purchases) were announced in March 2020.

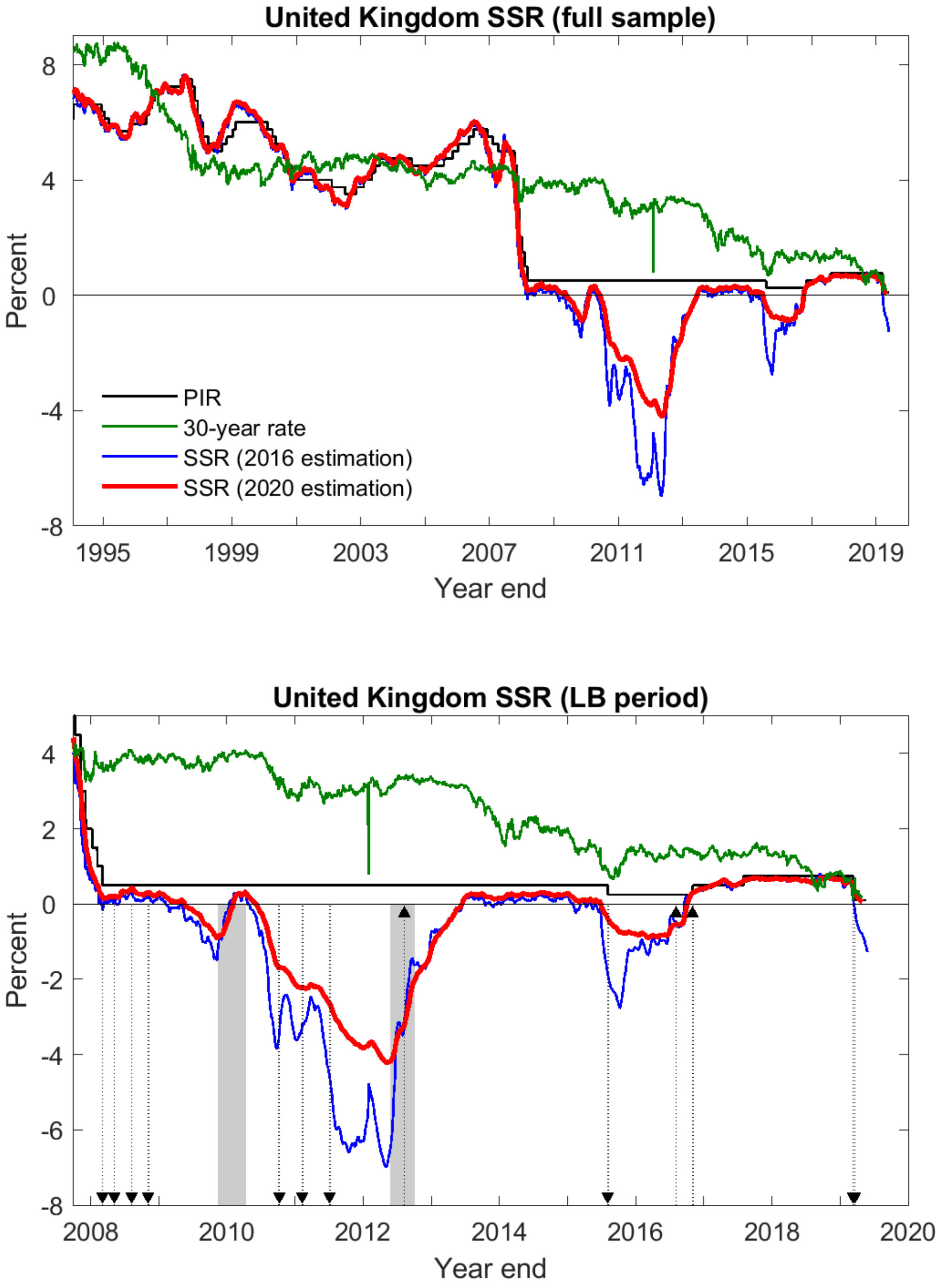

Figure 8: Panel 1 contains the old and new UK SSR estimates for the whole sample, along with the PIR and the 30-year interest rate. Panel 2 contains the same series in panel 1 but from September 2008, and also “down” and “up” arrows to respectively indicate major UMP easing and tightening events.

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A Details on model specification and estimation

This appendix details the K-ANSM(2) model used to produce the estimates on my website, and provides and overview of the yield curve data and estimation.

A.1 Model specification

K-ANSM(2) shadow short rates are:

$$\mathrm{r}\left(t\right) = L\left(t\right) + S\left(t\right) \tag{2}$$

where $L\left(t\right)$ and $S\left(t\right)$ are the Level and Slope state variables, respectively. The state variables under the physical $\mathbb{P}$ measure evolve as a correlated vector Ornstein-Uhlenbeck process:

$$\mathrm{d}x\left(t\right) = \kappa\left[\theta - x\left(t\right)\right]\,\mathrm{d}t + \sigma \mathrm{d}W\left(t\right) \tag{3}$$

where:

$$\begin{aligned} x_t &= \begin{bmatrix} L_t \\ S_t \end{bmatrix} ;\ \kappa = \begin{bmatrix} \kappa_{11} & \kappa_{12} \\ \kappa_{21} & \kappa_{22} \end{bmatrix} ;\ \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \\ \sigma &= \begin{bmatrix} \sigma_1 & 0 \\ \rho_{12}\sigma_2 & \sigma_2\sqrt{1-\rho_{12}^2} \end{bmatrix} \end{aligned} \tag{4}$$

and $\mathrm{d}W\left(t\right)$ is a 2×1 vector of independent Wiener increments.

K-ANSM(2) shadow forward rates, $\mathrm{f}(t,u)$, are:

$$\begin{aligned} \mathrm{f}\left(t,u\right) &= L\left(t\right) + S\left(t\right)\cdot\exp\left(-\phi u\right) \\ &\quad -\sigma_1^2\cdot\frac{1}{2}u^2 - \sigma_2^2\cdot\frac{1}{2}\left[G\left(\phi,u\right)\right]^2 - \rho_{12}\sigma_1\sigma_2\cdot uG\left(\phi,u\right) \end{aligned} \tag{5}$$

where:

$$G\left(\phi,u\right) = \frac{1}{\phi}\left[1 - \exp\left(-\phi u\right)\right] \tag{6}$$

K-ANSM(2) LB forward rates are:

$$\begin{aligned}\underline{\mathrm{f}}\,(t,u) &= \mathrm{r}_L + \left[\mathrm{f}\,(t,u) - \mathrm{r}_L\right] \cdot \Phi\left[\frac{\mathrm{f}\,(t,u) - \mathrm{r}_L}{\omega\,(u)}\right] \\ &\quad + \omega\,(u) \cdot \phi\left[\frac{\mathrm{f}\,(t,u) - \mathrm{r}_L}{\omega\,(u)}\right] \end{aligned} \tag{7}$$

where $\Phi\,[\cdot]$ is the cumulative unit normal probability density function, $\phi\,[\cdot]$ is the unit normal probability density function:[17]

$$\phi\,[\cdot] = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left[\frac{\mathrm{f}\,(t,u) - \mathrm{r}_L}{\omega\,(u)}\right]^2\right) \tag{8}$$

and $\omega\,(\tau)$ is:

$$\omega\,(u) = \sqrt{\sigma_1^2 \cdot u + \sigma_2^2 \cdot G\,(2\phi, u) + 2\rho_{12}\sigma_1\sigma_2 G\,(\phi, u)} \tag{9}$$

K-ANSM(2) interest rates, $\underline{\mathrm{R}}(t,\tau)$, are calculated from K-ANSM(2) forward rates using the standard term structure relationship:

$$\underline{\mathrm{R}}\,(t,\tau) = \frac{1}{\tau}\int_0^\tau \underline{\mathrm{f}}\,(t,u)\;\mathrm{d}u \tag{10}$$

which I evaluate by univariate numerical integration with rectangular increments.[18]

B Estimation method

The K-ANSM(2) underlying the new SSR estimates uses a time-varying LB value obtained from the CumMin mechanism discussed in section 3, i.e.:

$$\mathrm{r}_{\mathrm{LB},t} = \min\left\{12.5, [\text{actual PIR setting}]_t\right\} \tag{11}$$

For the remaining 10 parameters, i.e. $\mathbb{B} = \{\phi,\ \kappa_{11},\ \kappa_{12},\ \kappa_{21},\ \kappa_{22},\ \theta_1,\ \theta_2,\ \sigma_1,\ \sigma_2,\ \rho_{12}\}$, I impose the following constraints:

- the $\mathbb{P}$ measure mean-reversion matrix κ is specified to take the ANSM form, so the five parameters κ_{11}, κ_{12}, κ_{21}, κ_{22}, and θ_1 may be represented by a single parameter to be estimated. This is a technical change that greatly speeds up estimation, but does not alter the nature of the model and the SSR results relative to the fully parameterized K-ANSM(2).
- the Level and Slope volatility parameters σ_1 and σ_2 are selected to calibrate the SSR volatility to the PIR and 3-month interest rate data. As discussed in section 3, this change makes the evolution of SSR estimates smoother and less prone to short-lived spikes.

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[17] The parameter ϕ is completely unrelated to the function $\phi\,[\cdot]$. This is a coincidental collision of two standard notations.

[18] The integral therefore becomes a simple average of the sequence of $\underline{\mathrm{f}}(t,u)$ up to τ.

The resulting parameter set to be estimated is therefore reduced to from 10 parameters to four parameters.

I estimate the model using the iterated extended Kalman filter, which allows for the non-linearity of $\underline{\mathrm{R}}(t,\tau)$ with respect to the state variables. I prefer to use the iterated extended Kalman filter because it is acknowledged to be more reliable than the extended Kalman filter in general,[19] and I also found it to be more reliable when applied to estimating K-ANSMs; see Krippner (2015b).

The state equation for the K-ANSM(2) is a first-order vector autoregression:

$$x_t = \theta + \exp\left(-\kappa\Delta t\right)\left(x_{t-1} - \theta\right) + \varepsilon_t \tag{12}$$

where the subscripts t are an integer index to represent the progression of time in steps of Δt between observations (e.g. 1/12 for month-end data), $\exp\left(-\kappa\Delta t\right)$ is the matrix exponential of $-\kappa\Delta t$, and ε_t is the vector of innovations to the state variables. The variance of ε_t is:

$$\mathrm{var}\left[\varepsilon_t\right] = \int_0^{\Delta t} \exp\left(-\kappa u\right)\sigma\sigma' \exp\left(-\kappa' u\right)\,\mathrm{d}u \tag{13}$$

which is a 2×2 matrix.

The measurement equation for the K-ANSM(2) is:

$$\begin{bmatrix} \mathrm{R}_t\left(\tau_1\right) \\ \vdots \\ \mathrm{R}_t\left(\tau_K\right) \end{bmatrix} = \begin{bmatrix} \underline{\mathrm{R}}\left(x_t, \tau_1, \mathbb{B}\right) \\ \vdots \\ \underline{\mathrm{R}}\left(x_t, \tau_K, \mathbb{B}\right) \end{bmatrix} + \begin{bmatrix} \eta_t\left(\tau_1\right) \\ \vdots \\ \eta_t\left(\tau_K\right) \end{bmatrix} \tag{14}$$

where k is the index for the yield curve data of difference times to maturity τ_k, $\mathrm{R}_t\left(\tau_k\right)$ is the observed interest rate at time index t for the time to maturity τ_k, $\underline{\mathrm{R}}(x_t, \tau_k, \mathbb{B})$ are the K-ANSM(2) interest rate functions evaluated at τ_k, and $\eta_t\left(\tau_k\right)$ is the component of $\mathrm{R}_t\left(\tau_k\right)$ that is unexplained by the K-ANSM(2).

The measurement equation in vector form is:

$$\underline{\mathrm{R}}_t = \underline{\mathrm{R}}\left(x_t, \mathbb{B}\right) + \eta_t \tag{15}$$

where $\underline{\mathrm{R}}_t$, $\underline{\mathrm{R}}(x_t, \mathbb{B})$, and η_t are all $K \times 1$ vectors. I specify the variance of η_t to be a homoskedastic and diagonal, i.e.:

$$\Omega_\eta = \mathrm{diag}\left[\left\{\sigma_\eta^2, \ldots, \sigma_\eta^2\right\}\right] \tag{16}$$

where Ω_η is a $K \times K$ matrix with entries σ_η^2, and σ_η.[20] As also standard in the literature, I assume that the vectors η_t and ε_t are uncorrelated over time, and the covariances between η_t and ε_t are zero.

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[19] For example, Grewal and Andrews (2008) p. 312 cites Lefebvre, Bruyninckx, and De Schutter (2004) to note that the iterated extended Kalman filter outperforms the extended Kalman filter (and the unscented Kalman filter).

[20] A heteroskedastic specification could also be used, i.e. $\Omega_\eta = \mathrm{diag}\left[\left\{\left[\sigma_\eta\left(\tau_1\right)\right]^2, \ldots, \left[\sigma_\eta\left(\tau_K\right)\right]^2\right\}\right]$ where each of the parameters $\left[\sigma_\eta\left(\tau_k\right)\right]^2$ are allowed to be different. However, 11 additional parameters would be required, given the 12 maturities used for the estimation. In previous work, I also found that the That the heteroskedastic specification leads to larger variances in the residuals for the short- and long-maturity data, which has the practical effect of a less close fit to the short-maturity data and more volatile SSR estimates in both non-LB and LB periods. The homoskedastic specification enforces similar sized residuals across the yield curve data, which results in less volatile SSR estimates.

C Yield curve data

The datasets used for the each of the G4 countries are as follows:

- The sample periods are all from the start of January 1995 to the latest available daily data at the time of estimation (26 May 2020 at the time of writing).
- The maturities are 0.25, 0.5, 1, 2, 3, 4, 5, 10, 15, 20, and 30 years.
- The data are daily government interest rates spliced with overnight indexed swap (OIS) rates from when the full yield curve of data for the latter first become available. Note that the I use linear pro-rated values between the government and OIS data over the first year of splicing, to ensure against jumps in the data. Those OIS dates are 4 January 2006 for the United States, 28 May 2008 for the Euro Area, 6 August 2009 for Japan, and 30 May 2008 for the United Kingdom.

The datasets used for Switzerland, Canada, Australia, and New Zealand are as above, except government yield curve data is used for the entire sample.