

Documentation for measures of monetary policy

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1 Introduction

This documentation provides the essential background information on how to interpret the results provided on the website “Measures of the stance of United States monetary policy”, and how they are obtained. The documentation also applies to the results on the website “Comparison of international monetary policy measures”, which were added in January 2015 and differ only by the datasets used for their estimation.

Section 2 discusses the concepts of the Shadow Short Rate, the Expected Time to Zero, and the Effective Monetary Stimulus obtained from the K-ANSM(2). Sections 3, 4, and 5 contain further details on the K-ANSM(2) specification, estimation method, and yield curve datasets used for estimation. I note up front that results from shadow/lower bound term structure models will differ depending on the model specification and the data used for the estimation, particularly for Shadow Short Rates as I discuss in section 2.1. I therefore include brief explanations in this note on my particular choices, and relevant references.

Note also that the model specification and its estimation outlined in this documentation has changed from the previous version dated 30 September 2014, which remains available for reference. The new model has been used since 31 May 2016 to improve the Shadow Short Rates estimates from several perspectives. Appendix B contains the non-technical summary, provided at the time, of the changes and their benefits. A brief technical summary, with details provided in sections 3 to 5, is as follows. First, the model used to produce the estimates on the website is the K-ANSM(2), which is a two state-variable shadow yield curve model within the Krippner (2011, 2012b,c, 2013d,e, 2015) shadow/lower-bound (LB) framework. I previously used a K-ANSM(2) specified with an estimated LB parameter and heteroskedastic residuals, and estimated it using month-end data. I now use a K-ANSM(2) specified with an fixed lower bound parameter of 12.5 basis points and homoskedastic residuals, and estimate it using daily data. These changes result in Shadow Short Rates estimates that are: (1) less volatile; (2) more comparable between economies; and (3) available at a daily frequency.

Section 6 provides an overview of how to run the MatLab code, which is available from the website. This includes the code currently used to obtain the website results, and the original code used to obtain the results in my book Krippner (2015b). Related to the

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latter, appendix C details several (minor) errors and issues of which users of my book and original MatLab code have advised me. Finally, appendix A lists the major unconventional monetary policy events for the United States that are plotted in the figures on the website.

I welcome questions and comments regarding the information on the website, this note, and my related work. Please contact me at leo.krippner@rbnz.govt.nz. Also, please email me if you would like to be added to a distribution list informing you of any changes to the material on the websites and relevant updates of my research.

2 Overview of three monetary policy measures

The K-ANSM(2) readily provides three quantitative measures that can potentially be used as a quantitative indicator of the stance of monetary policy. In figure 1, I have illustrated examples of the **Shadow Short Rate (SSR)**, the **Expected Time to Zero (ETZ)**, and the **Effective Monetary Stimulus (EMS)** for an unconstrained and LB-constrained environments.

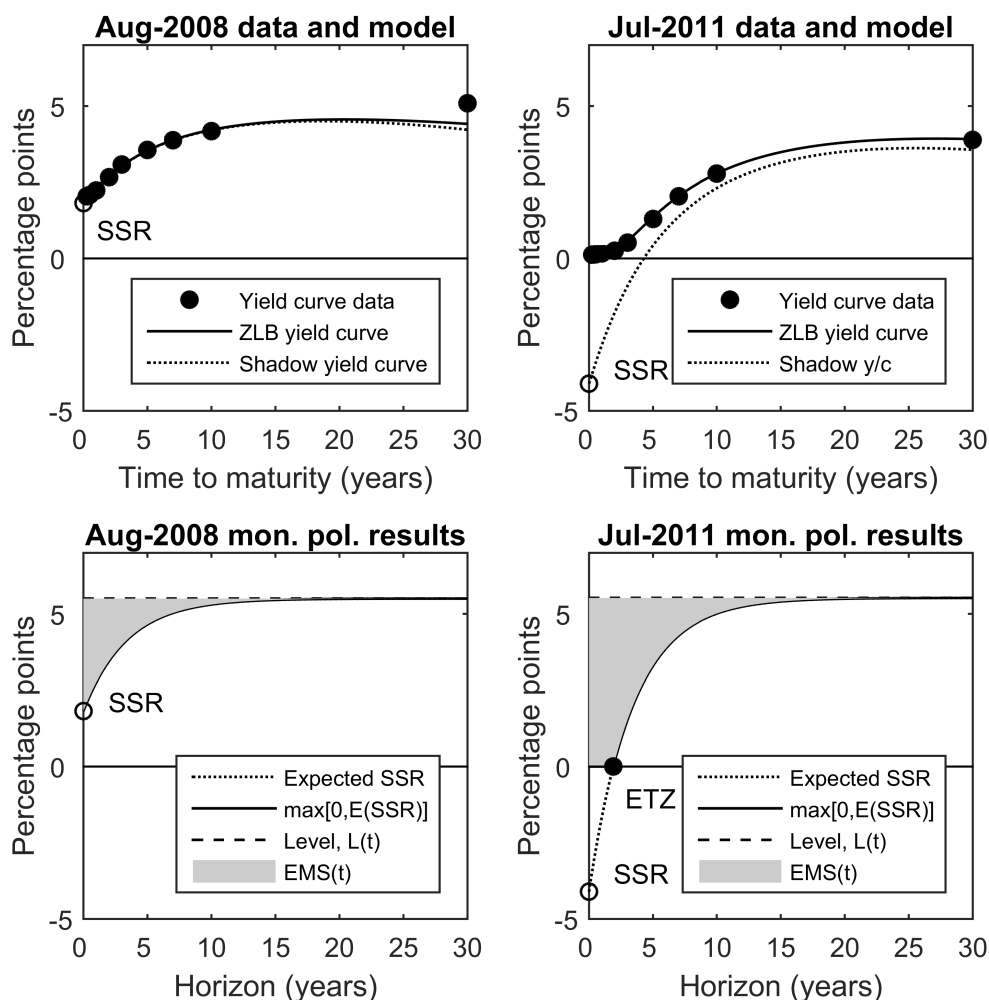


Figure 1: Examples of yield curve data and shadow/LB yield curve model estimates, and the associated SSR, the ETZ, and the EMS estimates. The August 2008 data and results are an example of a non-ZLB/conventional monetary policy environment, and July 2011 is a LB/unconventional monetary policy environment.

The following sub-sections summarize the SSR, ETZ and EMS monetary policy metrics, and sections 3 and 4 detail the K-ANSM(2) specification and estimation used to obtain the results available on the website.

2.1 Shadow Short Rate (SSR)

The SSR is the shortest maturity rate from the estimated shadow yield curve. It is essentially equal to the policy interest rate in non-LB/conventional monetary policy environments (e.g. August 2008), but the SSR can freely evolve to negative values in LB/unconventional environments (e.g. July 2011) to indicate an overall stance of policy that is more accommodative than a near-zero policy rate alone. In particular, the SSR reflects the effects that unconventional policy actions (such as quantitative easing and forward guidance) have on longer-maturity interest rate securities, because it is estimated from yield curve data.

SSRs have therefore become a popular and intuitive indicator of the stance of monetary policy across conventional and unconventional environments. Krippner (2011, 2012b,c, 2013b, 2015b), Bullard (2012, 2013), and Wu and Xia (2013, 2014, 2016) provide discussion on SSRs in that regard.

However, users should be aware of two issues when using SSRs, particularly for quantitative empirical applications. First, as discussed in Krippner (2014b,c and 2015b), an in-principle issue with SSRs is that negative values do not represent interest rates at which economic agents can transact. Therefore, the levels and changes in SSRs when they are negative should not necessarily be expected to influence the economy in the same way as policy rate levels and changes in conventional policy periods.

Second, the magnitude and profile of SSR estimates within unconventional periods vary, sometimes substantially, according to the model specification (particularly the LB parameter) and the data used for estimation. These points are illustrated in Bauer and Rudebusch (2016), Christensen and Rudebusch (2015), Krippner (2015b),¹ and are detailed in Krippner (2015a), most notably in reference to the Wu and Xia (2016) SSR estimates from a three-factor model. Bauer and Rudebusch (2016) and Christensen and Rudebusch (2015) therefore recommend not using any set of SSR estimates as a quantitative monetary policy indicator. I concur that three-factor SSR estimates should not be used in that regard; depending on the exact SSR series used, the variations in both magnitude and profile would likely lead to quite different interpretations about the stance of monetary policy and varying empirical results. In addition, Krippner (2015a) shows that three-factor SSR estimates do not correlate well with unconventional monetary policy events, and they sometimes produce counterintuitive positive values during unconventional periods.

However, the results for the United States in Krippner (2015a) indicate that SSR estimates from K-ANSM(2) models do provide useful quantitative indicators of unconventional monetary policy, and hence I think it is useful to retain them in the suite of unconventional monetary policy indicators. A brief summary of the results is as follows:

- Alternative estimates from different K-ANSM(2) models produce negative SSR estimates with similar profiles, i.e. they are ordinaly robust.

¹The original version of this documentation, which remains available on the website, contains the relevant results from Krippner (2015b).

- The negative SSR levels and changes are highly correlated with the evolution of unconventional monetary policy events.
- The negative SSR levels are highly correlated with lift-off metrics, such as the Expected Time to Zero in the following sub-section.
- The SSR series has negative levels similar to those obtained from the Taylor (1999) rule.

The ordinal robustness and noted correlations indicate that K-ANSM(2) SSR estimates are at least useful for monitoring whether monetary policy has become more or less accommodative compared to history. Nevertheless, the magnitudes of negative K-ANSM(2) SSR estimates inevitably remain subject to some sensitivity with respect to the model specification and the data used for estimation. Hence, for quantitative empirical applications, the Taylor (1999) rule provides a useful external calibration to justify the SSR series for the United States provided on the website. The model specification for the United States is also used for the other economies, which creates series that are most comparable between economies (but which does not necessarily produce the best fit to the data for each economy; also see footnote 7).

An alternative to using a single SSR series based on the approximate Taylor (1999) rule calibration noted above is to test the robustness of the result from any quantitative analysis to a range of alternative K-ANSM(2) SSR estimates. A simple way to proxy such alternative estimates is to up and down-scale the negative SSR values in the series from the website. Otherwise alternative estimates may be obtained using the MatLab programs supplied on the website.

2.2 Expected Time to Zero (ETZ)

If the SSR is negative, as in the July 2011 example, the ETZ indicates the future time horizon when the expected path of the SSR will reach zero. The expected path of the SSR is a simple function of the estimated state variables and parameters for the shadow/LB yield curve model, and so the ETZ may be readily calculated using those estimates.

Empirically, ETZ estimates are quite robust.² However, one practical drawback is that the ETZ does not provide a quantitative measure of monetary policy when the SSR is non-negative, such as for the August 2008 example in figure 1. Also, even when the SSR is negative, the ETZ doesn't account for the expected profile of the policy rate after it is expected to evolve above zero, which is likely to be an important consideration for economic agents. Nevertheless, the ETZ does provide a useful cross-check against changes in market expectations for policy rate "lift-off" from a prevailing near-zero setting.³

2.3 Effective Monetary Stimulus (EMS)

The EMS summarizes the current and expected path of the actual or LB-constrained short rate relative to an estimate of the steady-state/long-horizon nominal natural interest rate

²See the results from the original version of this documentation.

³The ETZ values provided on the website are under the risk-adjusted \mathbb{Q} measure, and so therefore include the effect of risk premiums. Hence, they will not in principle be directly comparable to surveyed lift-off horizon values; the latter are under the physical \mathbb{P} measure and so should not include any risk premium effects.

(LNIR). The EMS is obtained by calculating the total area between the expected path of the SSR truncated at zero and the LNIR proxy. For the August 2008 example in figure 1, the SSR and its expected path were all positive, so no truncation at zero is required to calculate the EMS. For the July 2011 example, the SSR and its expected path are negative out to the ETZ horizon, and those values are truncated to zero to calculate the EMS. The truncation represents that only the positive part of the SSR relative to the LNIR is effective for monetary stimulus, because the actual interest rates faced by economic agents cannot fall below zero.

In the current inception of the EMS on the website, the nominal natural rate is proxied by the Level state variable $L(t)$ estimated from the model. A higher EMS value indicates more stimulus (i.e. a larger and/or longer time of the expected policy rate below the LNIR).

Empirically, the EMS is quite robust, but it is dependent on the proxy used for the LNIR.⁴ As discussed in Krippner (2014b,c and 2015b), EMS measures are theoretically appealing because they are based on expected actual LB-constrained policy rates, and they are consistent and comparable across both non-LB and LB environments.

3 Model specification

The Krippner (2011, 2012b,c, 2013d,e, 2015) shadow/LB framework uses a continuous-time Gaussian affine term structure model (GATSM) to represent the shadow term structure, and the LB is imposed using a call option on shadow bonds with a strike price based on the lower bound for interest rates r_L (e.g. $r_L = 0$ gives a strike price of 1 for the shadow bond). The options reproduce the Black (1995) lower bound mechanism:

$$\underline{r}(t) = \max \{r_L, r(t)\} \tag{1}$$

where $\underline{r}(t)$ is the LB-constrained short rate, $r(t)$ is the shadow short rate, and $\max \{r_L, r(t)\}$ imposes the lower bound.

The model used to produce the estimates on the website uses an arbitrage-free Nelson and Siegel (1987) model with two state-variables (Level and Slope), or ANSM(2), to represent the shadow yield curve. I therefore call the associated shadow/LB model the Krippner ANSM(2), or K-ANSM(2). The reason for choosing an ANSM to represent the shadow yield curve is theoretical; Krippner (2012d, 2014a, 2014e, 2015) shows that ANSMs provide a parsimonious approximation to any GATSM that could be used regardless of its particular specification. The reason for choosing two factors is empirical; i.e. the ordinal robustness of two-factor SSR estimates as discussed in section 2.1.

K-ANSM(2) shadow short rates are:

$$r(t) = L(t) + S(t) \tag{2}$$

where $L(t)$ and $S(t)$ are the Level and Slope state variables, respectively. The state variables under the physical \mathbb{P} measure evolve as a correlated vector Ornstein-Uhlenbeck

⁴The original version of this documentation contains the results from Krippner (2015b). My work in progress suggests a more suitable LNIR proxy than the Level state variable $L(t)$, and also highlights the importance of accounting for risk premiums. As mentioned in Krippner (2015b), these alternative EMS estimates involve using data additional to yield curve data for the model estimation. Using the alternative proxy for the LNIR results in more accommodative EMS values than currently available on the website.

process:

$$dx(t) = \kappa [\theta - x(t)] dt + \sigma dW(t) \quad (3)$$

where:

$$\begin{aligned} x_t &= \begin{bmatrix} L_t \\ S_t \end{bmatrix} ; \kappa = \begin{bmatrix} \kappa_{11} & \kappa_{12} \\ \kappa_{21} & \kappa_{22} \end{bmatrix} ; \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \\ \sigma &= \begin{bmatrix} \sigma_1 & 0 \\ \rho_{12}\sigma_2 & \sigma_2\sqrt{1-\rho_{12}^2} \end{bmatrix} \end{aligned} \quad (4)$$

and $dW(t)$ is a 2×1 vector of independent Wiener increments.

K-ANSM(2) shadow forward rates are:

$$\begin{aligned} f(t, u) &= L(t) + S(t) \cdot \exp(-\phi u) \\ &\quad - \sigma_1^2 \cdot \frac{1}{2} u^2 - \sigma_2^2 \cdot \frac{1}{2} [G(\phi, u)]^2 - \rho_{12}\sigma_1\sigma_2 \cdot uG(\phi, u) \end{aligned} \quad (5)$$

where:

$$G(\phi, u) = \frac{1}{\phi} [1 - \exp(-\phi u)] \quad (6)$$

K-ANSM(2) LB forward rates are:

$$\begin{aligned} \underline{f}(t, u) &= r_L + [\underline{f}(t, u) - r_L] \cdot \Phi \left[\frac{\underline{f}(t, u) - r_L}{\omega(u)} \right] \\ &\quad + \omega(u) \cdot \phi \left[\frac{\underline{f}(t, u) - r_L}{\omega(u)} \right] \end{aligned} \quad (7)$$

where $\Phi[\cdot]$ is the cumulative unit normal probability density function, $\phi[\cdot]$ is the unit normal probability density function:⁵

$$\phi[\cdot] = \frac{1}{\sqrt{2\pi}} \exp \left(-\frac{1}{2} \left[\frac{\underline{f}(t, u) - r_L}{\omega(u)} \right]^2 \right) \quad (8)$$

and $\omega(\tau)$ is:

$$\omega(u) = \sqrt{\sigma_1^2 \cdot u + \sigma_2^2 \cdot G(2\phi, u) + 2\rho_{12}\sigma_1\sigma_2 G(\phi, u)} \quad (9)$$

K-ANSM(2) interest rates, $\underline{R}(t, \tau)$, are calculated from K-ANSM(2) forward rates using the standard term structure relationship:

$$\underline{R}(t, \tau) = \frac{1}{\tau} \int_0^\tau \underline{f}(t, u) du \quad (10)$$

which I evaluate by univariate numerical integration with rectangular increments.⁶

⁵The parameter ϕ is completely unrelated to the function $\phi[\cdot]$. This is a coincidental collision of two standard notations.

⁶The integral therefore becomes a simple average of the sequence of $\underline{f}(t, u)$ up to τ .

4 Estimation method

The K-ANSM(2) with a fixed lower bound has 10 free parameters to estimate, that is, $\mathbb{B} = \{\phi, \kappa_{11}, \kappa_{12}, \kappa_{21}, \kappa_{22}, \theta_1, \theta_2, \sigma_1, \sigma_2, \rho_{12}\}$. I set $r_L = 12.5$ basis points.⁷

I estimate the model using the iterated extended Kalman filter, which allows for the non-linearity of $\mathbf{R}(t, \tau)$ with respect to the state variables. I prefer to use the iterated extended Kalman filter because it is acknowledged to be more reliable than the extended Kalman filter in general,⁸ and I also found it to be more reliable when applied to estimating K-ANSMs; see Krippner (2013d,e).

The state equation for the K-ANSM(2) is a first-order vector autoregression:

$$x_t = \theta + \exp(-\kappa\Delta t)(x_{t-1} - \theta) + \varepsilon_t \quad (11)$$

where the subscripts t are an integer index to represent the progression of time in steps of Δt between observations (e.g. 1/12 for month-end data), $\exp(-\kappa\Delta t)$ is the matrix exponential of $-\kappa\Delta t$, and ε_t is the vector of innovations to the state variables. The variance of ε_t is:

$$\text{var}[\varepsilon_t] = \int_0^{\Delta t} \exp(-\kappa u) \sigma \sigma' \exp(-\kappa' u) du \quad (12)$$

which is a 2×2 matrix.

The measurement equation for the K-ANSM(2) is:

$$\begin{bmatrix} R_t(\tau_1) \\ \vdots \\ R_t(\tau_K) \end{bmatrix} = \begin{bmatrix} \mathbf{R}(x_t, \tau_1, \mathbb{B}) \\ \vdots \\ \mathbf{R}(x_t, \tau_K, \mathbb{B}) \end{bmatrix} + \begin{bmatrix} \eta_t(\tau_1) \\ \vdots \\ \eta_t(\tau_K) \end{bmatrix} \quad (13)$$

where k is the index for the yield curve data of difference times to maturity τ_k , $R_t(\tau_k)$ is the observed interest rate at time index t for the time to maturity τ_k , $\mathbf{R}(x_t, \tau_k, \mathbb{B})$ are the K-ANSM(2) interest rate functions evaluated at τ_k , and $\eta_t(\tau_k)$ is the component of $R_t(\tau_k)$ that is unexplained by the K-ANSM(2).

The measurement equation in vector form is:

$$\mathbf{R}_t = \mathbf{R}(x_t, \mathbb{B}) + \eta_t \quad (14)$$

where \mathbf{R}_t , $\mathbf{R}(x_t, \mathbb{B})$, and η_t are all $K \times 1$ vectors. I specify the variance of η_t to be a homoskedastic and diagonal, i.e.:

$$\Omega_\eta = \text{diag}[\{\sigma_\eta^2, \dots, \sigma_\eta^2\}] \quad (15)$$

⁷In the previous documentation, r_L was included in the parameter set to be estimated. However, further investigation has since shown that the magnitude of negative SSR estimates can be dominated by estimates of the LB parameter rather than general movements in the level and shape of the yield curve that the SSR is meant to reflect. The issue is compounded when the lower bound moves over time; for example the European Central Bank and the Bank of Japan policy rate settings have evolved to mildly negative levels. Using a fixed LB parameter for all economies provides estimated SSR series that are more comparable. However, the model will not necessarily fit the short-maturity data closely at all points in time, and the fixed LB parameter should not be taken as a literal indication of the LB that applies in practice at all points in time. Lemke and Vladu (2015) and Kortela (2015) explicitly model time variation in the lower bound for the euro-area, and such approaches are more appropriate if one requires a model with a lower bound that best matches the shorter-maturity data.

⁸For example, Grewal and Andrews (2008) p. 312 cites Lefebvre, Bruyninckx, and De Schutter (2004) to note that the iterated extended Kalman filter outperforms the extended Kalman filter (and the unscented Kalman filter).

where Ω_η is a $K \times K$ matrix with entries σ_η^2 , and σ_η .⁹ As also standard in the literature, I assume that the vectors η_t and ε_t are uncorrelated over time, and the covariances between η_t and ε_t are zero.

5 Yield curve data

I use daily yield curve data to estimate the K-ANSM(2).¹⁰ The following three points provide the essential description of the dataset used for the results on the website “Measures of the stance of United States monetary policy”:

- The sample period is 25 November 1985 to the latest available daily data at the time of estimation (noted in the spreadsheet updates). The start of the sample is determined by the availability of 30-year interest rate data from the Gürkaynak, Sack, and Wright (2007) data set noted below, but it also coincides with a consistent macroeconomic and policy period. Specifically, the disinflation period under Chairman Volker was completed so inflation was already relatively low and stable, the banking sector deregulation from the early 1980s had also been completed, and the primary monetary policy lever was the Federal Funds Target Rate (FFTR) over the entire period.¹¹
- The maturities are 0.25, 0.5, 1, 2, 3, 5, 10, and 30 years. These maturities are the standard benchmarks for Treasury notes and bonds from when the 30-year bond was first issued. I prefer to use the full maturity span of yield curve data, because the 30-year data should help to provide a better estimate of the Level component of the term structure than shorter maturity interest rates, which are subject to larger cyclical movements over the business cycle.
- The data are month-end government interest rates spliced with overnight indexed swap (OIS) rates, which obtains a long time series of data with the more-relevant OIS rates over the LB environment. The government interest rates are from the Gürkaynak, Sack, and Wright (2007) data set, up to December 2005. I have spliced those with Bloomberg overnight indexed swap (OIS) rate data from 4 January 2006, which is when the data set out to 30-years’ time to maturity first became available. I prefer to use OIS rates because they are directly relevant to expectations of the Federal Funds Rate. Note that I splice the government and OIS yield curve data using a linear pro-rated values of the government and OIS data over the first year

⁹In the previous documentation, the variance of η_t was specified as heteroskedastic and diagonal, i.e. $\Omega_\eta = \text{diag}\left\{\left\{[\sigma_\eta(\tau_1)]^2, \dots, [\sigma_\eta(\tau_K)]^2\right\}\right\}$. That specification leads to larger variances in the residuals for the short- and long-maturity data, which has the practical effect of a less close fit to the short-maturity data and more volatile SSR estimates in both non-LB and LB periods. The homoskedastic specification enforces similar sized residuals across the yield curve data, which results in less volatile SSR estimates.

¹⁰In the previous documentation, the parameters were obtained from a full estimation on a monthly frequency using end-of-month yield curve data. The daily SSR estimates were obtained using those monthly parameters and the daily yield curve data (which sometimes resulted in material differences between the monthly and daily SSR estimates). Now all of the results reported on the website are based on a full estimation at a daily frequency with daily data.

¹¹The Federal Open Market Committee only began making official FFTR announcements after meetings from 1992 but, prior to then, market participants could infer policy changes from open market operations.

in common,¹² which avoid any effects of discontinuities that could otherwise arise from the daily government and OIS rates being at different levels on a single splice day.

The datasets used for the United States, euro-area, Japan, and United Kingdom results on the website “Comparison of international monetary policy measures” are constructed similarly to the above.¹³ The key points are:

- The sample periods are all from 2 January 1995 to the latest available daily data at the time of estimation (noted in the spreadsheet updates). The start of the sample is determined by the availability of data from Bloomberg.
- The maturities are 0.25, 0.5, 1, 2, 3, 5, 10, and 30 years.
- The data are daily government interest rates spliced with overnight indexed swap (OIS) rates, with linear pro-rated values over the first year. The government interest rates are from Bloomberg, up to the day that reliable OIS rate data for the entire span of yield curve data out to 30 years become available. Those OIS dates are 4 January 2006 for the United States, 28 May 2008 for the euro-area, 6 August 2009 for Japan, and 30 May 2008 for the United Kingdom (previously 4 January 2006).

Note that the SSR series for the United States differs slightly from that available on the “Measures of the stance of United States monetary policy” website. The differences reflect the different data used to estimate the models underlying each series. I suggest using the longer SSR series for US-specific applications, and the shorter series for any comparisons between countries, although either series should give very similar results.

6 Running the MatLab code

This section provides an overview of the code and how to run it. Section 6.1 discussed the K-ANSM(2) code used since May 2016 to obtain the website updates, and section 6.2 discusses the code from the book. In both cases, as explained in section 6.3, the datasets I have used may be replicated provided one has access to the underlying yield curve data from Bloomberg. Alternatively, the “US_GSW_Govt.mat” dataset I have included may be used as an example. Of course, any other dataset with the format of “US_GSW_Govt.mat” may be used, or users can modify the code to suit their own customized datasets.

6.1 Code used from 31 May 2016

The code used since May 2016 to obtain the website updates is contained in the folder “B_NEW_KANSM2_20160531”. Brief documentation is contained within comments contained in the files. Users familiar with the original code will see that it remains similar in most respects, except for the changes noted in sections 3 and 4 and that I now use

¹²This is a change from the previous version of the documentation but, as noted on the Excel sheet “T. Record of changes”, it was actually introduced in the January 2016 update.

¹³The international SSR estimates are an addition to the previous version of the documentation. They were introduced on the website in January 2015.

separate files rather than using a switch in the code for running an update with given parameters or full estimations.¹⁴

The code is currently set to run an example with a historical version of the publicly available Gürkaynak, Sack, and Wright (2007) dataset. Hence:

- Running the file “AAA_RUN_UPDATE_KANSM2.m” will estimate the state variables and associated results using the parameters available in the file “US_GSW_Govt_rL125_Daily_20160708.mat”. It should reproduce the results contained in the file “US_GSW_Govt_rL125_Daily_20160708_UPDATE_Daily.mat”.
- Running the file “AAA_RUN_ESTIMATE_KANSM2” will undertake a full estimation of the model parameters and state variables (and associated results). The lower bound parameter can be estimated or set as a fixed parameters within the code. Note that the MatLab Optimization Toolbox is required for a full optimization.
- The results will be output in a separate “.mat” file and an Excel file, like the examples “US_GSWxls” contained in the folder.

The current SSR results on the “International comparisons” website may be replicated using the file “AAA_RUN_UPDATE_KANSM2.m”, the relevant parameters for each economy xx contained in “xx_BB_Govt_BB_OIS_rL125_Daily_20160429”, and by replicating the relevant datasets as discussed in section 6.3. The results on the “United States” website may be replicated using the parameters in the “US_GSW_Govt_BB_OIS_rL125_Daily_20160429.mat” file and the replicating the relevant dataset as discussed in section 6.3.

6.2 Original code

The original code used prior to May 2016 is contained in the folders “Book ...”. There are four folders, containing the K-ANSM(2) and K-ANSM(3) models, each with a fixed and an estimated lower bound. Brief documentation is contained within comments contained in the files.

The original code uses a switch for running an update with given parameters or full estimations, i.e.:

- Running “AAA_RUN_m” files with the setting “FinalNaturalParametersGiven=1;” will estimate the state variables using the parameters from “BOOK_ ... _X.mat” (or any alternative file specified by the user).
- Running “AAA_RUN_m” with the setting: “FinalNaturalParametersGiven=0;” will undertake a full estimation of parameters and state variables using the starting parameters from “BOOK_ ... _X.mat” (or any alternative file, or manual entries, as specified by the user). Note that the MatLab Optimization Toolbox is required for a full optimization.
- The results will be output in a separate “.mat” file and an Excel file.

¹⁴One further change in the code is that the numerical Hessian calculation is undertaken with much higher precision than previously; see the appendix section C.2 for further discussion.

To reproduce or update any of the six main results from Krippner (2015b) and/or the previous results available on the website, one will first need to reproduce the yield curve datasets as discussed in section 6.3. Alternatively, using “US_GSW_Govt.mat” will reproduce the results up to end-2005. The “BOOK_... X.mat” files in each folder contain the relevant parameters.

6.3 Dataset creation

The yield curve datasets that underlie the current estimates on the “United States” and “International comparisons” websites contain data that is proprietary to Bloomberg, which is why they not made available. Those with a Bloomberg subscription can use the files contained in the folder “A_NEW_ReadData_Files” to create the datasets as follows:

- Open the Excel spreadsheets “A_xx_All_Data_Bloomberg.xlsx” on a Bloomberg-enabled computer.
- Run the file “AA_A_Read_All_YC_CF_Data_RUN.m”.

If one wants to create the datasets used in Krippner (2015b) and the previous results on the “United States” and “International comparisons” websites, one can use the original data-reading files contained in the folder “Book_A_ReadDataFiles”:

- Open the Excel spreadsheets “A_xx_All_Data_Bloomberg.xlsx” on a Bloomberg-enabled computer.
- The files “AAA_Readm” and “AAB_Splicem” files will facilitate the reading and splicing of the data into a “.mat” file.

However, note that the splicing of government and OIS data from the original files occurs on a single day, rather the smooth pro-rated splicing over a year from the new data-reading files. Hence, even if one wants to use the original code for estimations and updates, I recommend applying the code to the newly created datasets.

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A List of US monetary policy events

The list below summarizes the dates of the announcements indicated in the US figures, along with my easing or tightening classification, and a brief description of the event itself. Note that I have also included other events that occurred during the same month as the main event, and sometimes I have combined close-by events to keep the indicators at a manageable number and distinct from each other within the figures.

1. Tuesday, December 16 2008 (easing): The FOMC end-of-meeting statement announced a 0 to 0.25 percent range for the FFTR, from the 1 percent target rate that had prevailed since the Wednesday, October 29 statement, effectively beginning the LB environment. Note that this date in the figures also captures the liquidity measures put in place by the Federal Reserve prior to December 16, in particular following the Monday, September 15 Lehmans’ bankruptcy. In addition, the first large scale asset purchase program announcement, the so-called “Quantitative Easing 1”, or QE1, was announced on Tuesday, November 25. QE1 amounted to purchases of \$1.725 trillion of mainly asset-backed securities up to when it ended in March 2010.
2. Friday, August 27 2010 (easing): FOMC Chairman Bernanke foreshadowed “Quantitative Easing 2”, or QE2, at a speech in Jackson Hole. QE2 was subsequently introduced on Wednesday, November 3 2010, and amounted to purchases of \$0.6 trillion of US Treasuries up to when it ended in June 2011. Another influence during this month was the Tuesday, August 10 FOMC statement that acknowledged a slowing of the economy.
3. Tuesday, August 9 2011 (easing): The FOMC statement announced the first explicit extended calendar forward guidance for the FFTR, with a conditional expectation that it would remain near zero to mid-2013. Another influence during this month was Bernanke’s announcement on Friday, August 26 that the upcoming September 21 FOMC meeting would be extended to two days to allow a fuller discussion of the range of tools that could be used for additional monetary stimulus. I have combined this announcement indicator with an announcement in the following month:

- Wednesday, September 21 2011 (easing; not indicated for clarity): The FOMC statement announced the maturity extension program, the so-called “Operation Twist”. Operation Twist was initially a \$0.4 billion program to sell shorter maturity Treasury securities and buy longer-term Treasury securities, but the Wednesday, June 20 2012 FOMC statement announced its extension and it ultimately amounted to \$0.67 trillion when it ended in late 2012.
4. Wednesday, January 25 2012 (easing): The FOMC statement announced an extension of the calendar forward guidance to late-2014.
 5. Thursday, September 13 2012 (easing): The FOMC statement announced an extension of the calendar forward guidance to mid-2015 and the introduction of “Quantitative Easing 3”, or QE3. QE3 was an open-ended program to purchase \$40 billion of asset-backed securities per month.
 6. Wednesday, December 12 2012 (easing): The FOMC statement announced a change from calendar forward guidance to guidance based on an unemployment rate of 6.5 percent. At the same meeting, QE3 was increased to \$85 billion purchases per month by adding \$45 billion of longer-term Treasury securities.
 7. Wednesday, May 22 2013 (tightening): Chairman Bernanke foreshadowed the potential tapering of QE3 at a congressional testimony on the economic outlook. I have combined this announcement indicator with an announcement in the following month.
 8. Wednesday, December 18 2013 to Wednesday October 29 2014 (tightening): The FOMC statement announced the first reduction of QE3 on the former date, and announced the final reduction to zero purchases on the latter date.

B Changes to the Shadow Short Rate estimates (31 May 2016)

The following is a non-technical overview of the changes to the model, and hence results, provided with the May 2016 website updates.

This month’s update contains international Shadow Short Rate (SSR) estimates that, apart from the United States, are distinctly different in magnitude from those previously reported. The differences reflect a combination of changes I have made to the model and its estimation, which in turn improve the SSR estimates from several perspectives.

In brief, the changes and their implications are:

- The lower bound has been fixed at 12.5 basis points for all estimations, which results in SSR series that are more comparable between economies. Previously I estimated the lower bound separately for each economy, but different lower bound estimates can dominate the magnitudes of the estimated SSR series over changes to the yield curve data.
- The model residuals are now specified to have equal variances, which generally results in lower volatility for the SSR series. Previously I allowed the variances to

differ, which resulted in a less close fit to the short-maturity data and higher SSR volatilities.

- The model is now estimated using daily data, which results in no discrepancies between the SSR estimates at different frequencies (because all are based off the estimated daily SSR series). Previously I estimated the model with monthly data, and the daily SSRs estimated with the monthly parameters sometimes differed very materially from the monthly SSR estimates.

The figure on the following page provides a comparison of the previous and the new SSR estimates, with the confidence intervals for the previous SSRs.

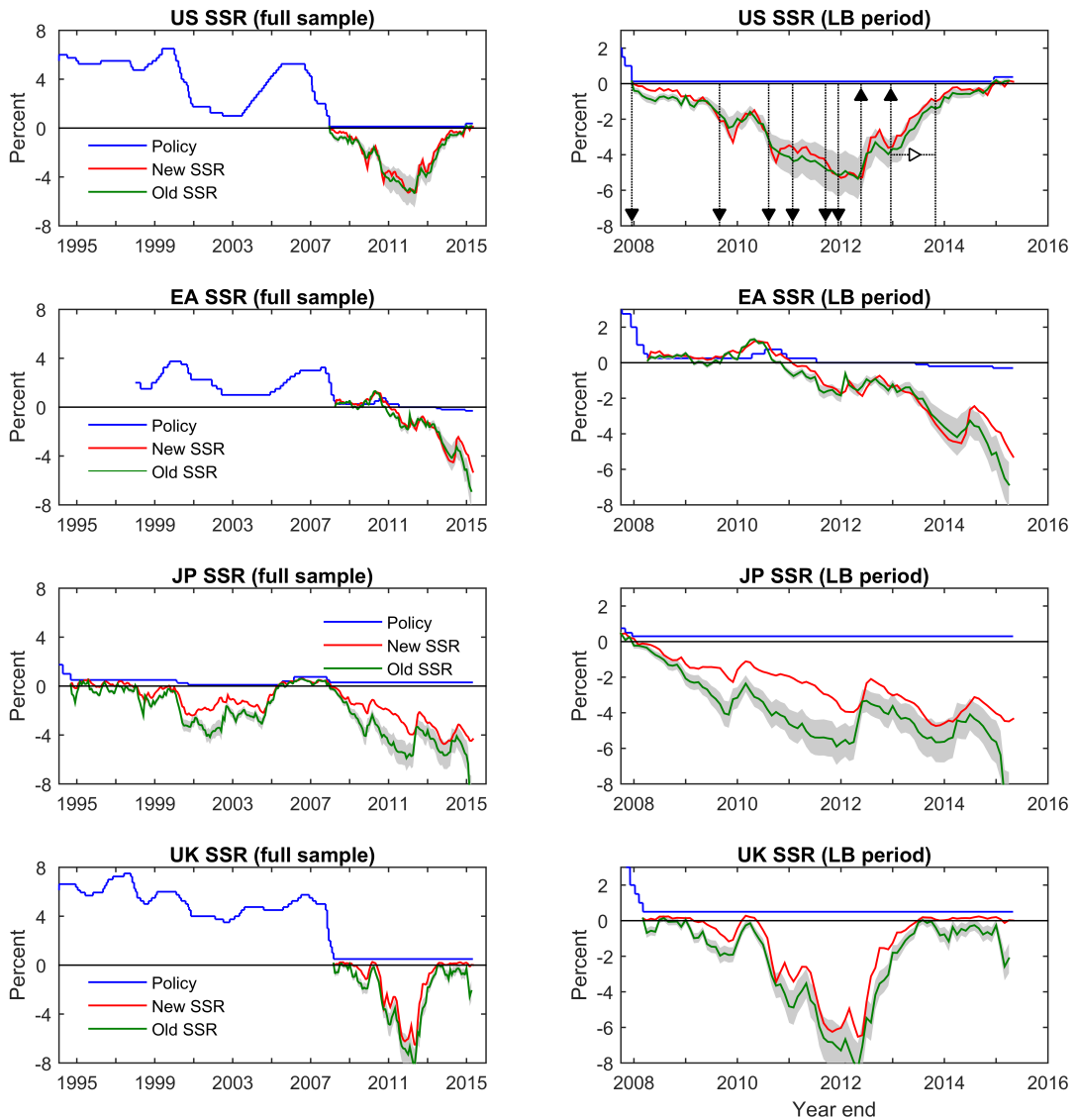
The material changes to some of the SSR series provide a timely reminder that SSR estimates unavoidably vary with the model and data used to estimate them. The documentation and my working paper “A comment on Wu and Xia (2015) and the case for two-factor shadow short rates”, both available on the “Measures of the stance of United States monetary policy” webpage, highlights that issue and references the related work of others in that regard. The issue is particularly acute for three-factor models, which includes Wu and Xia (2016), because the associated SSR estimates are not robust in profile or magnitude.

Nevertheless, the working paper also provides evidence that two-factor SSR estimates provide useful indicators of the stance of monetary policy over conventional and unconventional periods, which is why I continue to make them available. In brief:

- The negative SSR levels and changes correlate intuitively with unconventional monetary policy events.
- The negative SSR levels have a similar magnitudes to levels obtained from the Taylor (1999) rule.
- Alternative estimates from different specifications of the same class of model produce negative SSR estimates with similar profiles, so they are ordinally robust.

The magnitudes of two-factor SSR estimates are subject to some sensitivity. Hence, ideally any quantitative empirical application of them should include robustness checks with a series that up- and down-scales the negative SSR values. Alternatively, one could use an externally motivated calibration, like the Taylor (1999) rule, to scale the magnitudes of the negative values.

In summary, the range of potential indicators for the stance of monetary policy when policy rates are constrained by the lower-bound is still in development, and their empirical applicability is still being tested. Unless or until a leading candidate is obtained, SSR estimates obtained from two-factor shadow/lower-bound models, while certainly not perfect in all respects, have many favorable properties and deserve to retain a place in the suite of unconventional monetary policy indicators.



C Book and code corrections

This section details several (minor) corrections and issues of which users of my book and/or MatLab code have advised me. I thank Sander Muns and Eric McCoy for bringing these issues to my attention (respectively, sections 6.1 and 6.2, and section 6.3). I have myself noticed typographical errors in the book, and I intend to provide a list of corrections in due course. However, those errors are readily apparent from the context.

C.1 Corrections to the estimation procedure tables

There are two discrepancies between the expressions contained in the estimation procedure tables in the book and the associated MatLab code. The code is correct in both cases,

and I apologize for any confusion that the discrepancy in the book may have caused to other readers.

The expression $P_t^- = FP_{t-1}^+F' + \Theta(\Delta t)$ that occurs in step 3.1 of table 4.2 of the book (and elsewhere) should be $P_t^- = FP_{t-1}^+F' + V\Theta(\Delta t)V'$. The expression $x_{t,i+1}^+ = x_{t,i}^+ + K_{t,i}\eta_{t,i}$ that occurs near the end of the iteration loop in step 3.2 should be $x_{t,i+1}^+ = x_t^- + K_{t,i}\eta_{t,i}$. For reference, the lines of code corresponding to these expressions are, respectively:

- “P_Minus=F*P_Plus*F'+Q;” within the IEKF recursion loop in the “AAC_EKF_CAB_GATSM_SingleLoop” function, where the quantity “Q” is $V\Theta(\Delta t)V'$ (Q remains constant over the IEKF recursions, so it is calculated once prior to the recursions); and
- “x_Plus_i1=x_Minus+K_i*w_i;”.

Below is a corrected version of table 4.2, and the list of all of the places in the book where the errors should be corrected.

Table 4.2Partial K-AGM estimation via IEKF

1. Setup and estimation constraints:Calculate F , $\Theta(\Delta t)$, and Ω_η **2. State initialization:**

$$x_0^+ = \theta$$
$$P_0^+ = V\Theta(\infty)V'$$

3. Recursion:for $t = 1 : T$ **3.1. Prior state estimates:**

$$x_t^- = \theta + F(x_{t-1}^+ - \theta)$$
$$P_t^- = FP_{t-1}^+F' + V\Theta(\Delta t)V'$$

3.2. Measurement/posterior state iterations:Set : $x_{t,0}^+ = x_t^-$; $H_{t,0} = 0$ **ITERATE** : from $i = 0$

$$\eta_{t,i} = \mathbb{R}_t - \mathbb{R}(x_{t,i}^+, \mathbb{A}) - H_{t,i}(x_t^- - x_{t,i}^+)$$

$$H_{t,i} = \left. \frac{\partial}{\partial x(t)} \mathbb{R}[x(t), \mathbb{A}] \right|_{x(t)=x_{t,i}^+}$$

$$M_{t,i} = H_{t,i}P_t^-H_{t,i}' + \Omega_\eta$$

$$K_{t,i} = P_t^-H_{t,i}'M_{t,i}^{-1}$$

$$x_{t,i+1}^+ = x_t^- + K_{t,i}\eta_{t,i}$$

EXIT : at max(i) or $|x_{t,i+1}^+ - x_{t,i}^+|$ tolerance**3.3. Posterior state estimates:**

$$x_t^+ = x_{t,i+1}^+, \eta_t = \eta_{t,i}, \text{ and } M_t = M_{t,i}$$

$$P_t^+ = (I - K_{t,i}H_{t,i}')P_t^-$$

Record x_t^+ , η_t , and M_t next t

Notes:

 x_t^- , x_t^+ , $x_{t,i}^+$, and θ are $N \times 1$ vectors $\eta_{t,i}$ and η_t are $K \times 1$ vectors κ_D is an $N \times N$ diagonal matrix P_t^- , P_t^+ , $\sigma\sigma'$, $\Theta(\Delta t)$, and $\Theta(\infty)$ are $N \times N$
symmetric matrices V and F are $N \times N$ asymmetric matrices $M_{t,i}$ and M_t is a $K \times K$ symmetric matrix Ω_η is a $K \times K$ diagonal matrix $K_{t,i}$ is an $N \times K$ matrix $H_{t,i}$ is a $K \times N$ matrix

The precursor expressions are originally introduced correctly in equations 3.9 and 3.24. The first instance of the error is the expression for P_t^- in table 3.1. The error was subsequently transcribed into tables 3.2, 4.3, and 5.3, and 5.4. The fully worked example

based on table 3.2 is also in error. It should be:

$$\begin{bmatrix} P_{11,t}^- & P_{12,t}^- \\ P_{12,t}^- & P_{22,t}^- \end{bmatrix} = F \begin{bmatrix} P_{11,t-1}^+ & P_{12,t-1}^+ \\ P_{12,t-1}^+ & P_{22,t-1}^+ \end{bmatrix} F' + V \begin{bmatrix} \Theta_{11}(\Delta t) & \Theta_{12}(\Delta t) \\ \Theta_{12}(\Delta t) & \Theta_{22}(\Delta t) \end{bmatrix} V^{-1} \quad (16)$$

and equation 3.99 should be:

$$P_t^- = F P_{t-1}^+ F' + V \Theta(\Delta t) V' \quad (17)$$

Unrelated to the above, equation 3.97 should be:

$$P_t^- = V \begin{bmatrix} \Sigma_{11} \frac{1}{2\kappa_1} & \Sigma_{12} \frac{1}{\kappa_1 + \kappa_2} & \Sigma_{13} \frac{1}{\kappa_1 + \kappa_3} \\ \Sigma_{12} \frac{1}{\kappa_1 + \kappa_2} & \Sigma_{22} \frac{1}{2\kappa_2} & \Sigma_{23} \frac{1}{\kappa_2 + \kappa_3} \\ \Sigma_{13} \frac{1}{\kappa_1 + \kappa_3} & \Sigma_{23} \frac{1}{\kappa_2 + \kappa_3} & \Sigma_{33} \frac{1}{2\kappa_3} \end{bmatrix} V' \quad (18)$$

C.2 Estimation of standard errors via the Hessian

The standard errors obtained using the numerically calculated Hessian can vary according to the step size used in that procedure (if the MatLab defaults from the code it is based on are over-riden with smaller values). To rectify this, I now calculate the standard errors using a more accurate calculation of the Hessian. Specifically, the Hessian procedure now uses the Richardson extrapolation to obtain numerical derivatives, which has substantially smaller orders of error than the MatLab-based code.¹⁵ However, the calculation takes longer to implement, particularly because I have specified a high degree of accuracy (which users may relax if they wish). I intend to add an outer product gradient estimation of parameter standard errors in the future.

C.3 Correction of parameter indexing within interim estimations

The interim update step of the MatLab script “AAA_RUN_KANSM2_Est_LB” within the “C_KANSM2_Estimated_LB” folder, contains the incorrect indexing of some parameters. The following lines are the corrected versions (which I have already changed in the code):

```
Line 137: InitialParameters(11)=fzero(@(x)x/(1+abs(x))...
-InitialNaturalParameters(11),1);
Line 160: KappaQ=[0,0;FinalNaturalParameters(2),0];
Line 161: KappaP=[FinalNaturalParameters(3),FinalNaturalParameters(4);...
FinalNaturalParameters(5),FinalNaturalParameters(6)];
```

Note that this error did not affect the final parameter estimates. It only affected the recording of interim updates, where the parameters are occasionally saved before being subsequently being used as new starting values. However, because those starting values were recorded incorrectly, it may have slowed the convergence for full estimations.

¹⁵My code is based on that obtained from “<http://www.cs.tut.fi/~hakkin22/censored/richardson.m>”.